

Section 2.4: Applications of Linear Systems

Tuesday, October 03, 2023 4:33 PM

handout question #1: given points $(1, -2)$, $(-1, 8)$, $(2, -1)$,
find

$$y = ax^2 + bx + c$$

answer: plug in points into equation

$$(1, -2): \quad y = ax^2 + bx + c$$

$$-2 = a(1)^2 + b(1) + c$$

$$-2 = a + b + c$$

$$(-1, 8) \quad 8 = a(-1)^2 + b(-1) + c$$

$$8 = a - b + c$$

$$(2, -1) \quad -1 = a(2)^2 + b(2) + c$$

$$-1 = 4a + 2b + c$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 1 & -1 & 1 & 8 \\ 4 & 2 & 1 & -1 \end{array} \right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$a = 2$$

$$b = -5$$

$$c = 1$$

$$y = ax^2 + bx + c$$

$$\boxed{y = 2x^2 - 5x + 1}$$

handout question #2:

in L v = ... 1 ... in = a ...

let $x =$ amount of ingredient 1
 $y =$ 2
 $z =$ 3

magnesium : $10x + 30y + 20z = 120$
 Vitamin C : $20x + 50y + 30z = 220$
 calcium : $60x + 130y + 70z = 620$

$$\left[\begin{array}{ccc|c} 10 & 30 & 20 & 120 \\ 20 & 50 & 30 & 220 \\ 60 & 130 & 70 & 620 \end{array} \right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} x - z = 6 \\ y + z = 2 \end{array}$$

↑
let $z = t$

$$\begin{cases} x = t + 6 \\ y = 2 - t \\ z = t \end{cases}$$

now, cannot have negative amounts

$$\begin{array}{lll} x \geq 0 & t + 6 \geq 0 & t \geq -6 \\ y \geq 0 & 2 - t \geq 0 & t \leq 2 \\ z \geq 0 & t \geq 0 & t \geq 0 \end{array}$$

$$0 \leq t \leq 2$$

soln is

$$\begin{cases} x = t + 6 \\ y = 2 - t \\ z = t \end{cases} \text{ for } 0 \leq t \leq 2$$

what if the RREF for this word problem looked like:

$$y - \frac{1}{3}z = 0$$

so let $z = t$

$$\begin{cases} w = \frac{2}{3}t \\ x = \frac{1}{2}t \\ y = \frac{1}{3}t \\ z = t \end{cases}$$

now pick a value of t that gives us the smallest set of positive integers

let $t = 6$

$$\begin{aligned} \text{so } w &= \frac{2}{3}t = 4 \\ x &= 3 \\ y &= 2 \\ z &= 6 \end{aligned}$$



handout question #4: network analysis

for each node (intersection), traffic in = traffic out

$$\begin{aligned} A: & 10 + 10 = f_1 + f_2 \\ B: & f_1 + f_3 = 20 + 5 \\ C: & 15 + 15 = f_3 + f_4 \\ D: & f_2 + f_4 = 15 + 10 \end{aligned} \quad \begin{cases} f_1 + f_2 = 20 \\ f_1 + f_3 = 25 \\ f_3 + f_4 = 30 \\ f_2 + f_4 = 25 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 1 & 0 & 25 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 1 & 0 & 1 & 25 \end{array} \right]$$

REF \rightarrow

let $f_4 = t$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{cases} f_1 = t - 5 \\ f_2 = -t + 25 \\ f_3 = -t + 30 \\ f_4 = t \end{cases}$$

↑
solution to system

b) $f_4 = t = 10$ so $\begin{cases} f_1 = 5 \\ f_2 = 15 \\ f_3 = 20 \end{cases}$

c) we want all of $f_1, f_2, f_3,$ and $f_4 \geq 0$

$$\begin{array}{lcl} \text{so} & t - 5 \geq 0 & \rightarrow t \geq 5 \\ & -t + 25 \geq 0 & 25 \geq t \\ & -t + 30 \geq 0 & \underline{30 \geq t} \\ & t \geq 0 & \underline{t \geq 0} \end{array}$$

so $5 \leq t \leq 25$

when $t = 5$

$$\begin{array}{l} f_1 = 0 \\ f_2 = 20 \\ f_3 = 25 \\ f_4 = 5 \end{array}$$

when $t = 25$

$$\begin{array}{l} f_1 = 20 \\ f_2 = 0 \\ f_3 = 5 \\ f_4 = 25 \end{array}$$

so

	min	max
f_1	0	20
f_2	0	20
f_3	5	25
f_4	5	25

question 5 from handout

- find all possible combinations of 20 coins (nickels, dimes, quarters) that will make exactly \$3.

$$\text{nickel} = 5¢$$

$$\text{dime} = 10¢$$

answer: $n + d + q = 20$

$$5n + 10d + 25q = 300$$

$$\left[\text{or } 0.05n + 0.10d + 0.25q = 3 \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 5 & 10 & 25 & 300 \end{array} \right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -20 \\ 0 & 1 & 4 & 40 \end{array} \right]$$

so $n - 3q = -20$
 $d + 4q = 40$

↑
let $q = t$

$$\begin{cases} n = 3t - 20 \\ d = -4t + 40 \\ q = t \end{cases}$$

BUT want all of $n, d,$ and q to be ≥ 0 and integer

$$3t - 20 \geq 0$$

$$40 - 4t \geq 0$$

$$t \geq 0$$

so $t \geq 6\frac{2}{3}$ → round to 7

$$t \leq 10$$

$$t \geq 0$$

then $7 \leq t \leq 10$

and integer

$$\begin{cases} n = 3t - 20 \\ d = -4t + 40 \\ q = t \end{cases} \quad \text{for } t = 7, 8, 9, 10$$

totally optional and possibly annoying chef:

$$t = 7$$

$$n = 1$$

$$d = 12$$

$$q = 7$$

$$\begin{array}{r} 5 \text{ ¢} \\ 120 \text{ ¢} \\ 175 \text{ ¢} \\ \hline 300 \text{ ¢} \end{array}$$

