Review for Test 2:

Tuesday, October 30, 2018 4:30 PM

covers section 2.3 to 3.4, inclusive

Section 2.3:

$$\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3 \dots \vec{v}_k) = \{c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots + c_k\vec{v}_k\}$$

the set of all vectors you can make out of linear combos of $\vec{v}_1$ through $\vec{v}_k$

- to check if $\vec{u}$ is in span$(\vec{v}_1, \vec{v}_2 \dots \vec{v}_k)$, see if $\vec{u}$ is a linear combo of $\vec{v}_1 \to \vec{v}_k$

check RREF $\left[\begin{array}{ccc|c} \vec{v}_1 & \vec{v}_2 & \dots & \vec{v}_k & \vec{u} \end{array}\right]$

has at least one solution

linear independence

- if $c_1\vec{v}_1 + c_2\vec{v}_2 + \dots c_k\vec{v}_k = 0$ only has the unique trivial solution $c_1 = c_2 = c_3 = \dots c_k = 0$, then vectors are LI

otherwise, LD

example: determine whether

$$\vec{u} = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

are LI or LD

answer: $\begin{bmatrix} 1 & 2 & 3 \\ -5 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ or $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -5 & 2 & 1 & 0 \\ 2 & 1 & 2 & 0 \end{array}\right]$

RREF $\downarrow$

$\begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & 4/3 \\ 0 & 0 & 0 \end{bmatrix}$

LD

note: $\vec{w} = \frac{1}{3}\vec{v} + \frac{4}{3}\vec{u}$

Section 2.4:

word problems

- set up system
- RREF
- give solutions
    - give max/min values for variables when there is more than one solution

Sections 3.1 & 3.2:

matrices

A^T swaps rows $\leftrightarrow$ columns of A

$(A^T)^T = A$

matrix inverses:

2x2: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\uparrow$
ad - bc

in general: $[A \mid I] \rightsquigarrow [I \mid A^{-1}]$

to solve $A^{-1}A\vec{x} = A^{-1}\vec{b}$

$\vec{x} = A^{-1}\vec{b}$

properties of inverses:

$(A^{-1})^{-1} = A$

$(AB)^{-1} = B^{-1}A^{-1}$

$(A^n)^{-1} = (A^{-1})^n$

$(A^T)^{-1} = (A^{-1})^T$

solving equations:

solve for X:

$BX = (BA)^{-1}$

$B^{-1}BX = B^{-1}(BA)^{-1}$

$X = B^{-1}A^{-1}B^{-1}$

$B^{-1}B = I$

Section 3.3

elementary matrices

if $I \xrightarrow{\text{one row op}} E$

then $A \xrightarrow{\text{same row op}} B$

then $B = EA$

Section 3.4:

LU factorization

A

row operations

U

want zeros

shortcut:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ & 1 & 0 & 0 \\ & & 1 & 0 \\ & & & 1 \end{bmatrix}$$

zeros above the main diagonal

U

want zeros below main diagonal

L

these entries are from "undoing" the row ops on A

ones on the main diagonal

What do you do with it?

$$A\vec{x} = \vec{b}$$

$$\underbrace{LU\vec{x}}_{\vec{y}} = \vec{b}$$

solve $L\vec{y} = \vec{b}$ first

then put $\vec{y}$ into

$$\vec{y} = U\vec{x}$$