

Review for Test 3

Friday, November 25, 2022 10:53 AM

subspace: contains $\vec{0}$ and all sums and scalar multiples of its vectors

basis of S :

a basis is a set of vectors that spans the subspace S and is LI

dimension (S) = number of vectors in the basis

example: to show that $\vec{v}_1, \vec{v}_2, \vec{v}_3 \dots \vec{v}_n$ is a basis for \mathbb{R}^n , ← these are column vectors

$$\begin{bmatrix} | & | & | & \dots & | \\ v_1 & v_2 & v_3 & \dots & v_n \\ | & | & | & \dots & | \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} | & | & | & \dots & | \end{bmatrix}$$

if vectors are row vectors, then transpose them to get columns

↑
if no free variables, then it's a basis

Row(A) = row space of A
= space made up of the rows of A

basis - take non-zero rows from RREF

problem: if you want the row space in terms of the original rows of A , then transpose A to make rows into columns, use column space approach, transpose back

Col(A) = column space

basis \rightarrow RREF matrix A
note the columns with leading ones in the RREF, take corresponding columns from A itself

Null(A) = the set of all \vec{x} for which

$$A\vec{x} = \vec{0}$$

Rank(A) = number of leading variables in RREF
 Nullity(A) = " " free " " "

$$\text{RREF of } A = \begin{bmatrix} \textcircled{1} & 0 & 1 & 0 & 5 \\ 0 & \textcircled{1} & 3 & 0 & 7 \\ 0 & 0 & 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\uparrow \quad \uparrow \quad \uparrow$

$$\left. \begin{array}{l} \text{Rank}(A) = 3 \\ \text{Nullity}(A) = 2 \end{array} \right\} \text{sum} = \# \text{ columns}$$

change of basis:

$$\text{basis: } \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \}$$

want vector $\vec{0}$ in terms of this basis

$$\left[\begin{array}{c|c|c|c} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{0} \end{array} \right] \rightsquigarrow \text{RREF}$$

Section 3.6:

transformations -

projection operator will be given if needed
 rotation matrix " " " " " "

- you could be asked to find the matrix that corresponds to a projection / rotation / reflection

- evaluate for given vectors

- first column of A is the effect of that transformation onto \hat{i}

- second is \hat{j}

- I can also ask for A for
- reflection

- for more than one transformation, order matters

first T , then S
↑ ↑
matrix A matrix B

$$(S \circ T)\vec{x} = S(T(\vec{x})) = BA\vec{x}$$

- also, transformations are linear so

$$T(a\vec{x} + b\vec{y}) = aT(\vec{x}) + b(T\vec{y})$$

Complex numbers:

$$z = a + bi \quad (\text{rectangular})$$

$$\begin{aligned} z &= r(\cos \theta + i \sin \theta) && (\text{trig}) \\ &= r e^{i\theta} && (\text{Euler/exponential}) \\ &= r \angle \theta && (\text{phasor}) \end{aligned} \quad \left. \vphantom{\begin{aligned} z &= r(\cos \theta + i \sin \theta) \\ &= r e^{i\theta} \\ &= r \angle \theta \end{aligned}} \right\} \text{polar}$$

- addition/subtraction $(3+5i) + (2-7i)$

- multiplication/division $(3+5i)(2-7i)$ ← just FOIL

$$\frac{3+5i}{2-7i} \left(\frac{2+7i}{2+7i} \right) \quad \leftarrow \text{multiply by complex conjugate}$$

$$\frac{2e^{i\pi}}{4e^{-5\pi/6i}} \quad \leftarrow \text{use exponent rules}$$

- to raise to a power:

$$(2e^{i\pi})^9 = 2^9 e^{i9\pi}$$

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- to find complex roots:

↑
may have to
find coterminal
 $0 \leq \theta < 2\pi$

$$\text{if } z^n = r \angle \theta$$

$$\text{then } z = r^{1/n} \angle \left[\frac{\theta}{n} + \frac{k360^\circ}{n} \right] \text{ for } k = \underbrace{0, 1, 2, \dots, n-1}_{n \text{ of them}}$$

note: eigenvalues/eigenvectors can be complex

Chapter 4:

Determinants of square matrices

3 special cases:

2x2

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

3x3

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad \begin{array}{l} \text{cross} \\ \text{product} \\ \text{method} \end{array}$$

triangular matrix of
any size

- product of main
diagonal

in general:

method of minors / cofactor expansion

expand along top row \Rightarrow

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = +a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} \quad \underline{\underline{\text{etc}}}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Cofactor definition of A^{-1}

- will not test this directly, but it can be used to find an inverse matrix if you like

Cramer's Rule: if $A\vec{x} = \vec{b}$ where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

then $x_i = \frac{\det(A_i(\vec{b}))}{\det(A)}$

the matrix obtained by replacing column i of A by \vec{b}

Eigenvalues: def: if $A\vec{v} = \lambda\vec{v}$ for $v \neq \vec{0}$
then λ is an eigenvalue and \vec{v} is its eigenvector

to find λ : solve $\det(A - \lambda I) = 0$

- you will get a polynomial in λ
- factor it (if doesn't factor, use quad. formula)

- the power on each factor gives the algebraic multiplicity

for each λ , to find the eigenvector,

solve $(A - \lambda I)\vec{x} = \vec{0}$

REF

- the dimension of the eigenspace
= number of free variables in REF
= number of eigenvectors for that λ
= geometric multiplicity

(geo mult \leq alg mult)

note: eigenvalues and eigenvectors may be complex

Diagonalization: $A = PDP^{-1}$

where $D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$

and $P = [\vec{v}_1 | \vec{v}_2 | \dots | \vec{v}_n]$

λ_n has eigenvector \vec{v}_n

A is diagonalizable if and only if
 alg mult = geom mult for every λ

convenient application of $A = PDP^{-1}$

$$A^m = P D^m P^{-1} = P \begin{bmatrix} \lambda_1^m & & & \\ & \lambda_2^m & & \\ & & \ddots & \\ & & & \lambda_n^m \end{bmatrix} P^{-1}$$

