

Exploration: The Cross Product

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the cross product:

$$\text{in } \mathbb{R}^3 \quad \vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\text{then } \vec{u} \times \vec{v} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

the shortcut: recall unit vectors $\hat{i}, \hat{j}, \hat{k}$

then you could say, for example,

$$\vec{u} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\begin{aligned} \text{then } \vec{u} \times \vec{v} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ u_1 & u_2 & u_3 & u_1 & u_2 \\ v_1 & v_2 & v_3 & v_1 & v_2 \end{vmatrix} \end{aligned}$$

multiply through the lines
 add $\backslash\backslash$, then subtract $\//\//$

example: compute $\vec{u} \times \vec{v}$ for $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & | & \hat{i} & \hat{j} \\ 1 & 2 & -1 & | & 1 & 2 \\ 2 & 3 & 1 & | & 2 & 3 \end{vmatrix}$$

$$= \hat{i}(2)(1) + \hat{j}(-1)(2) + \hat{k}(1)(3) - \hat{k}(2)(2) - \hat{i}(-1)(3) - \hat{j}(1)(1)$$

$$= 2\hat{i} - 2\hat{j} + 3\hat{k} - 4\hat{k} + 3\hat{i} - \hat{j}$$

$$= 5\hat{i} - 3\hat{j} - \hat{k} = \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix}$$