# Section 1.2: cont'd:

Tuesday, September 11, 2018 4:27 PM

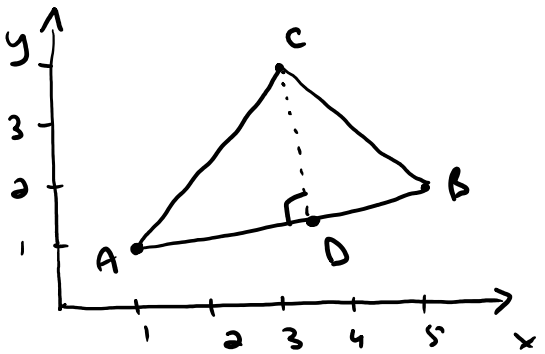
example: find $\text{proj}_{\vec{u}}(\vec{v})$ for:

$$\vec{u} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\text{proj}_{\vec{u}}(\vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\,\vec{u} \qquad \vec{u}\cdot\vec{v} = 3, \quad \vec{u}\cdot\vec{u} = 1$$

$$= \frac{3}{1}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

example: consider the following diagram with $A = (1,1)$, $B = (5,2)$, and $C = (3,4)$. Find point D.




we want the projection onto $\vec{AB}$ of $\vec{AC}$

$$\vec{AB} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \qquad \vec{AC} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\vec{AD} = \text{proj}_{\vec{AB}}(\vec{AC}) = \frac{\vec{AC}\cdot\vec{AB}}{\vec{AB}\cdot\vec{AB}}\,\vec{AB}$$

$$\vec{AC}\cdot\vec{AB} = 11$$

$\vec{AC} \cdot \vec{AB} = 11$

$\vec{AB} \cdot \vec{AB} = 17$

$$= \frac{11}{17}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\vec{D} = \vec{A} + \vec{AD}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 44/17 \\ 11/17 \end{bmatrix}$$

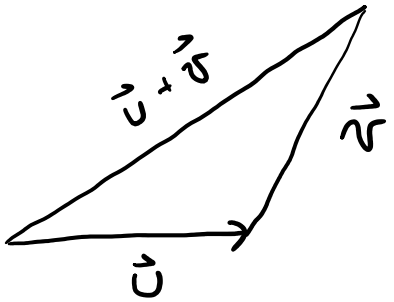
$$= \begin{bmatrix} 61/17 \\ 28/17 \end{bmatrix}$$

so point D is $\left(\frac{61}{17}, \frac{28}{17}\right)$

$$\approx (3.59, 1.64)$$

---

the Pythagorean theorem can be generalized to $\mathbb{R}^n$




$$\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v}) \cdot (\vec{u}+\vec{v})$$

$$= \vec{u} \cdot (\vec{u}+\vec{v}) + \vec{v} \cdot (\vec{u}+\vec{v})$$

$$= \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$$

$$= \vec{u}\cdot\vec{u} + 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$$

this statement is true for any vectors $\vec{u}$ and $\vec{v}$

but what if $\vec{u} \perp \vec{v}$? then $\vec{u}\cdot\vec{v} = 0$

$$\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$$