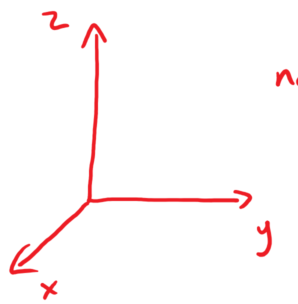
Cross-product, cont'd

Thursday, September 13, 2018   12:54 PM

example: Find a unit vector parallel to the y-z plane that that is perpendicular to $\vec{w}$, where

$$\vec{w} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$



note: a vector that is parallel to y-z plane has $x = 0$

method #1: use cross product

let $\vec{u}$ = vector we want

since $\vec{u}$ is $/\!/$ to yz-plane

then $\vec{u} \perp$ x-axis

$\vec{u} \perp \hat{i}$

and since $\vec{u} \perp \vec{w}$ then

$\vec{u}$ will be $/\!/$ to $\vec{w} \times \hat{i}$

$$\vec{w} \times \hat{i} = \left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & 0 \end{matrix}\right| \begin{matrix} \hat{i} & \hat{j} \\ 3 & -1 \\ 1 & 0 \end{matrix}$$

$= 2\hat{j} + \hat{k}$ ← not a unit vector

unit vector

$$\vec{u} = \pm \frac{\vec{w} \times \hat{i}}{\|\vec{w} \times \hat{i}\|} = \pm \frac{1}{\sqrt{5}} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\vec{u} = \pm \frac{\vec{w} \times \hat{i}}{\|\vec{w} \times \hat{i}\|} = \pm \frac{1}{\sqrt{5}} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

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method #2: dot product

$\vec{u}$ is // to y-z plane, so its x-coord is 0

let $\vec{u} = \begin{bmatrix} 0 \\ a \\ b \end{bmatrix}$ $\qquad \vec{w} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

want $\vec{u} \perp \vec{w}$ so $\vec{u} \cdot \vec{w} = 0$

$$\vec{u} \cdot \vec{w} = 0 - a + 2b = 0$$
$$a = 2b$$

but $\vec{u} = \begin{bmatrix} 0 \\ 2b \\ b \end{bmatrix}$

but $\vec{u}$ is a unit vector

$$\|\vec{u}\| = 1 = \sqrt{4b^2 + b^2}$$
$$1 = \sqrt{5b^2}$$
$$\frac{1}{5} = b^2 \qquad \text{and} \qquad b = \pm \frac{1}{\sqrt{5}}$$

$$\vec{u} = \begin{bmatrix} 0 \\ \pm 2/\sqrt{5} \\ \pm 1/\sqrt{5} \end{bmatrix}$$

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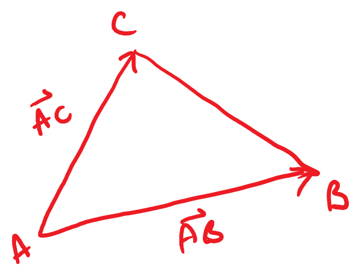
example: Find a vector $\vec{N}$ $\perp$ to the plane containing the points A, B, C where

$A = (-4, 0, 2)$, $B = (1, -3, 1)$, and $C = (2, -2, 6)$.



$$\vec{AB} = \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix} \quad \text{and} \quad \vec{AC} = \begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix}$$

$$\vec{N} = \vec{AB} \times \vec{AC} = \left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 5 & -3 & -1 \\ 6 & -2 & 4 \end{array} \right| \begin{array}{cc} \hat{i} & \hat{j} \\ 5 & -3 \\ 6 & -2 \end{array}$$

$$= -12\hat{i} - 6\hat{j} - 10\hat{k} + 18\hat{k} - 2\hat{i} - 20\hat{j}$$

$$\vec{N} = -14\hat{i} - 26\hat{j} + 8\hat{k}$$

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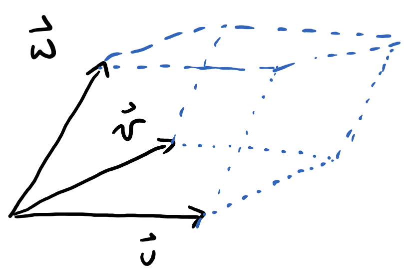
one more usage:



$$\text{volume} = \left| \vec{w} \cdot (\vec{u} \times \vec{v}) \right|$$

absolute value because volumes are non-negative