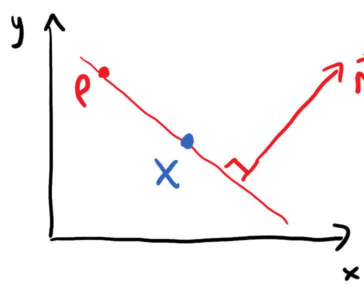
Section 1.3: cont'd

Friday, September 14, 2018 1:20 PM

example: find the general equation of the line perpendicular to $\vec{N} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, passing through the point $P = (1, 4)$.



$X = (x, y)$ point on the line

$\vec{PX} = \begin{bmatrix} x-1 \\ y-4 \end{bmatrix}$

$\vec{PX} \perp \vec{N}$

$\vec{PX} \cdot \vec{N} = 0$

normal form $\rightarrow$ $\begin{bmatrix} x-1 \\ y-4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 0$

$2(x-1) + 3(y-4) = 0$

general form $\rightarrow$ $\boxed{2x + 3y = 14}$

---

note: the general form looks like:

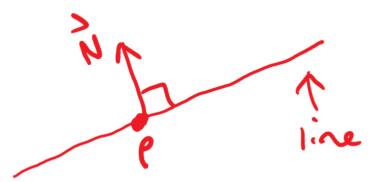
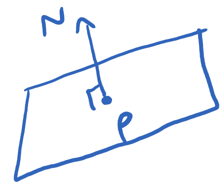
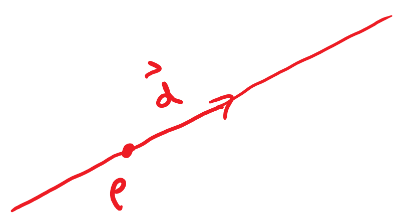
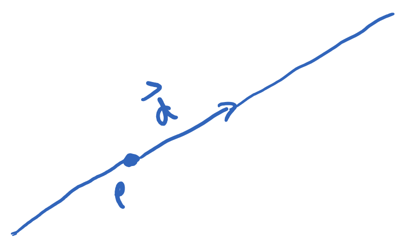
$Ax + By = C$ (or $Ax + By + C = 0$)

where A, B, C real and, if possible, integers with A positive in $\mathbb{R}^2$, can also define a line by a point P and a direction vector $\vec{d}$ that is parallel to the line

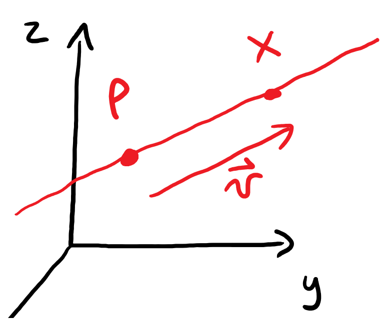
the nice thing is that this also works in $\mathbb{R}^3$

note: a point and a normal vector in $\mathbb{R}^3$ define a <u>plane</u>

$\mathbb{R}^2$ vs $\mathbb{R}^3$:

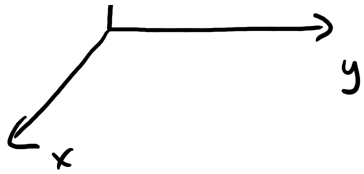
	$\mathbb{R}^2$	$\mathbb{R}^3$
point and normal vector		
point and parallel vector		

lines in $\mathbb{R}^3$:



given $P = (x_0, y_0, z_0)$

and vector $\vec{v}$ parallel to the line, where



the line, where

$$\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

let $X = (x, y, z)$ be an arbitrary point on the line

so $\vec{PX} \parallel \vec{v}$

$\vec{PX} = t\vec{v}$ where t = a scalar

$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} = t \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

vector equation of a line

$$\left\{ \begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \\ z &= z_0 + tc \end{aligned} \right.$$

parametric equations of a line in $\mathbb{R}^3$

grouping bracket

example: find parametric equations for the line joining points $P = (1, 4, -2)$ and $Q = (2, 3, 5)$

$$\vec{PQ} = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$$

could use $\vec{QP}$

could use $\vec{QX}$ here

$\vec{PX} = t\,\vec{PQ}$

$$\begin{bmatrix} x - 1 \\ y - 4 \\ \end{bmatrix} = t \begin{bmatrix} 1 \\ -1 \\ \end{bmatrix}$$

could use $\vec{QP}$ instead

$$\begin{bmatrix} x-1 \\ y-4 \\ z+2 \end{bmatrix} = t \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$$

$$\begin{cases} x = 1 + t \\ y = 4 - t \\ z = -2 + 7t \end{cases}$$

note: parametric equations of a line are not unique, since you could use Q as the point instead, or $\vec{QP}$ as vector, etc

---