

Section 1.3: cont'd

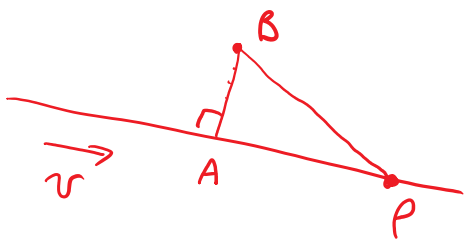
Wednesday, September 19, 2018 11:27 AM

from yesterday:

example: find the distance between the point $B = (1, -1, 2)$ and the line

$$\begin{cases} x = 3 + t \\ y = -2 - 2t \\ z = 4 + 2t \end{cases}$$

answer:



find a point P on the line

$$P = (3, -2, 4)$$

the direction vector $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$$\vec{PA} = \text{proj}_{\vec{v}}(\vec{PB})$$

then $\vec{AB} = \vec{PB} - \vec{PA}$

distance $d = \|\vec{AB}\|$

$$\vec{PB} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$$

$$\vec{PA} = \text{proj}_{\vec{v}}(\vec{PB}) = \frac{\vec{v} \cdot \vec{PB}}{\vec{v} \cdot \vec{v}} \vec{v}$$

$$= \left(\frac{-2-2-4}{1+4+4} \right) \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$= -\frac{8}{9} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$\vec{AB} = \vec{PB} - \vec{PA} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix} - \left(-\frac{8}{9}\right) \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10/9 \\ -7/9 \\ -2/9 \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} 10 \\ 7 \\ 2 \end{bmatrix}$$

$$d = \|\vec{AB}\| = \frac{1}{9} \sqrt{10^2 + 7^2 + 2^2}$$

$$= \frac{1}{9} \sqrt{153} = \frac{\sqrt{17}}{3} \approx 1.374$$