Section 2.3: cont'd

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recall: a set of vectors is LD if

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots c_k\vec{v}_k = \vec{0}$$

and at least one of the constants $c_1, c_2, c_3 \dots c_k$ is non-zero.

example:

$$2\underset{\vec{u}}{\begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}} + 3\underset{\vec{v}}{\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}} = \underset{\vec{w}}{\begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}}$$

these three vectors are linearly dependent (LD) because

$$2\vec{u} + 3\vec{v} = \vec{w}$$

$$2\vec{u} + 3\vec{v} - \vec{w} = \vec{0}$$

at least one non-zero constant

$$c_1\underset{\vec{u}}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} + c_2\underset{\vec{v}}{\begin{bmatrix} 2 \\ 3 \end{bmatrix}} = \vec{0}$$

$\vec{u}$ and $\vec{v}$ are LI

because $0\cdot\vec{u} + 0\vec{v} = \vec{0}$ is the only solution

example: Determine if the following sets of vectors are LD or LI. If they are LD, find the relationship between them.

a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

answer: is there a non-trivial solution to

$$c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \ ?$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 3 & 4 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$$

the only solution is $c_1 = c_2 = 0$

$\boxed{LI}$

b) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 4 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

LD

$$\begin{cases} c_1 + 3c_3 = 0 \\ c_2 - 2c_3 = 0 \end{cases}$$

free variable

let $c_3 = t$

$$\begin{cases} c_1 = -3t \end{cases}$$

$$\begin{cases} c_1 = -3t \\ c_2 = 2t \\ c_3 = t \end{cases}$$

or $$-3t\begin{bmatrix}1\\2\\0\end{bmatrix} + 2t\begin{bmatrix}1\\1\\-1\end{bmatrix} + t\begin{bmatrix}1\\4\\2\end{bmatrix} = 0$$

c) $$\begin{bmatrix}2\\1\\3\\5\end{bmatrix}, \begin{bmatrix}3\\2\\4\\1\end{bmatrix}, \begin{bmatrix}3\\1\\7\\1\end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 3 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 4 & 7 & 0 \\ 5 & 1 & 1 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$c_1 = c_2 = c_3 = 0$$

$$\therefore \boxed{LI}$$

theorem: let $\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots \vec{v}_k$ be vectors in $\mathbb{R}^n$, and let

$$A = [\vec{v}_1 \mid \vec{v}_2 \mid \vec{v}_3 \mid \dots \; \vec{v}_k]$$

then $\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots \vec{v}_k$ are LI if and only if

$[A \mid 0]$ has only the trivial solution

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theorem: any set of $\underbrace{M \text{ vectors}}_{\text{number of columns}}$ in $\underbrace{\mathbb{R}^n}_{\text{number of rows}}$ is LD

if $\boxed{M > n}$

this follows from Section 2.2
homogeneous and rows < columns
=> infinitely many solutions

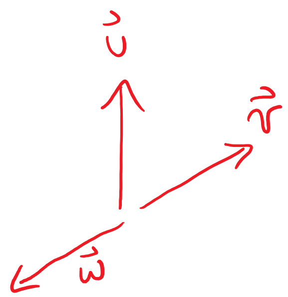
example: suppose $\{\vec{u}, \vec{v}, \vec{w}\}$ is LD.

Can we assume that $\vec{u}$ must be a linear combination of $\vec{v}$ and $\vec{w}$?

answer: $c_1 \vec{u} + c_2 \vec{v} + c_3 \vec{w} = 0$

what about $0\vec{u} + 2\vec{v} + \vec{w} = 0$?

$\vec{w} = -2\vec{v}$



we can't assume that $\vec{u}$ is a linear combo of $\vec{v}$ and $\vec{w}$, as in the above counterexample