Section 3.3: cont'd

Monday, October 15, 2018 11:23 AM

we looked at the Fundamental Theorem of Matrices from the handout

example: Find A if

a) $(I_2 + 2A)^{-1} = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$

$$\left((I_2 + 2A)^{-1}\right)^{-1} = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}^{-1}$$

$$I_2 + 2A = \frac{1}{2(4)-6(1)} \begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1/2 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & -3 \\ -1/2 & 1 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I_2}$$

$$2A = \begin{bmatrix} 1 & -3 \\ -1/2 & 0 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 1 & -3 \\ -1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & -3/2 \\ -1/4 & 0 \end{bmatrix}$$

b) $(2A^T)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$$2A^T = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^T = \frac{1}{}\begin{bmatrix} 7 & -2 \end{bmatrix}$$

$$A^{T} = \frac{1}{2}\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 7/2 & -3/2 \\ -1 & 1/2 \end{bmatrix}$$

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example: Solve for X, assuming all matrices are invertible

$$B^2 A^2 X A^{-1} B^{-1} = I$$

$$\underbrace{B^{-2} B^2}_{I} A^2 X A^{-1} B^{-1} = B^{-2} I$$

$$A^{-2} A^2 X A^{-1} B^{-1} = A^{-2} B^{-2}$$

$$X A^{-1} B^{-1} B = A^{-2} B^{-2} B$$

note: $B^{-2} B = B^{-1} \underbrace{B^{-1} B}_{I}$

$$X A^{-1} A = A^{-2} B^{-1} A$$

$$X = A^{-2} B^{-1} A$$

optional check:

$$B^2 A^2 X A^{-1} B^{-1} = I$$

$$B^2 \boxed{A^2 \quad A^{-2}} B^{-1} \boxed{A \quad A^{-1}} B^{-1} = I$$

$$B^2 B^{-1} B^{-1} = I \quad \checkmark$$