Section 3.5: cont'd

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example: let $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 2 & -4 \end{bmatrix}$.

is $\vec{w} = [3 \quad 2]$ in Row A?

answer: method #1

recall: Row A = span([1, -2], [0, 1], [2, -4])

if $\left[\begin{array}{c} A \\ \hline \vec{w} \end{array}\right] \xrightarrow{\text{row ops}} \left[\begin{array}{c} B \\ \hline 0 \end{array}\right]$, then $\vec{w}$ is in Row(A)

row ops except that we CANNOT move the bottom row

NO SWAPPING!

$$\begin{bmatrix} A \\ \vec{w} \end{bmatrix} = \left[\begin{array}{cc} 1 & -2 \\ 0 & 1 \\ 2 & -4 \\ \hline 3 & 2 \end{array}\right] \xrightarrow{R_4 - 3R_1} \left[\begin{array}{cc} 1 & -2 \\ 0 & 1 \\ 2 & -4 \\ \hline 0 & 8 \end{array}\right] \xrightarrow{R_4 - 8R_2} \left[\begin{array}{cc} 1 & -2 \\ 0 & 1 \\ 2 & -4 \\ \hline 0 & 0 \end{array}\right]$$

Yes, $\vec{w}$ is in Row(A)

method #2: use the RREF of $[A^T \mid \vec{w}^T]$

$$[A^T \mid \vec{w}^T] = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ -2 & 1 & -4 & 2 \end{array}\right]$$

$$\Big\downarrow \text{ RREF}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 8 \end{array}\right]$$

↑ free variable   infinite solutions

yes, $\vec{w}$ is in Row(A)

theorem: if 2 matrices are row equivalent, then they have the same row space

ie if $A \xrightarrow{\text{row ops}} B$

then Row(A) = Row(B)

definition: $\text{Null}(A) = \{ \vec{x} \text{ in } \mathbb{R}^N \mid A\vec{x} = \vec{0} \}$

is the null space of an $M \times N$ matrix A

theorem: $N = \text{Null}(A)$ is a subspace of $\mathbb{R}^N$

i) must include the origin

$$A\vec{x} = \vec{0}$$

↑ if $\vec{x} = 0$, does this work?

$$A\vec{0} = \vec{0}$$

ii) suppose $\vec{u}$ and $\vec{v}$ are in N

then $A\vec{u} = 0$
$A\vec{v} = 0$

if $A(\vec{u}+\vec{v})$ in N?

$$A(\vec{u}+\vec{v}) = A\vec{u} + A\vec{v} = \vec{0} + \vec{0} = \vec{0} \checkmark$$

iii) suppose $\vec{u}$ is in N, so $A\vec{u} = \vec{0}$

then $A(c\vec{u}) = c(A\vec{u})$
$= c\,\vec{0}$
$= \vec{0}$ $\checkmark$

definition: A basis of a subspace S of $\mathbb{R}^n$ is a set of vectors in S such that

i) the span of the vectors in the basis is S

ii) the vectors in the basis are linearly independent LI

note: I like to Think of a basis as the minimum number of vectors needed for spanning the space also: if S is a subspace of $\mathbb{R}^n$ and $\{\vec{v}_1, \vec{v}_2, \vec{v}_3 \dots \vec{v}_k\}$ is a basis for S, then for every vector $\vec{u}$ in S, there is exactly one way to write $\vec{u}$ as a linear combination of basis vectors $\vec{v}$

example: $\{\hat{i}, \hat{j}, \hat{k}\}$ is a basis of $\mathbb{R}^3$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{\hat{i}} + y\underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{\hat{j}} + z\underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\hat{k}}$$

$$\text{span}(\hat{i}, \hat{j}, \hat{k}) = \mathbb{R}^3$$

is $\{\hat{i}, \hat{j}, \hat{k}\}$ LI?

$$a\hat{i} + b\hat{j} + c\hat{k} = 0$$

has $a = b = c = 0$, the trivial solution