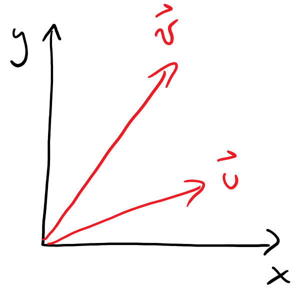
Section 3.5: cont'd

Friday, October 19, 2018 1:34 PM

example:



suppose $\vec{u}$ and $\vec{v}$ are in $\mathbb{R}^2$ and $\vec{u}$ and $\vec{v}$ are <u>not</u> scalar multiples of each other (not parallel)

then $\vec{u}$ and $\vec{v}$ are LI and span $(\vec{u}, \vec{v})$ is $\mathbb{R}^2$

so $\{\vec{u}, \vec{v}\}$ is a basis for $\mathbb{R}^2$

example: let $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 3 \\ 2 \\ 9 \end{bmatrix}$

find a basis of span $(\vec{u}, \vec{v}, \vec{w})$.

answer: check to see if $\vec{u}, \vec{v}, \vec{w}$ are LI

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 0 & 2 & 0 \\ 3 & -3 & 9 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variable

so infinitely many solutions

$\therefore$ LD

but the column is a recipe:

$\vec{w} = 2\vec{u} + (-1)\vec{v}$

since $\vec{w}$ is a linear combo of $\vec{u}$ and $\vec{v}$,

$$\text{span}(\vec{u}, \vec{v}, \vec{w}) = \text{span}(\vec{u}, \vec{v})$$

note: $\text{span}(\vec{u}, \vec{w})$ or $\text{span}(\vec{v}, \vec{w})$ would also work

$$\boxed{\text{basis is } \{\vec{u}, \vec{v}\}}$$

definition: the dimension of a subspace $S \neq \{\vec{0}\}$ is the number of vectors in a basis of S

the dimension of $S = \{\vec{0}\}$ is zero.

examples: the subspaces of $\mathbb{R}^3$

i) point $\{\vec{0}\}$ has dimension 0
ii) line through $\vec{0}$: $\text{span}\{\vec{v}\}$ where $\vec{v} \neq 0$
   - this subspace has dimension 1
iii) plane through $\vec{0}$: $\text{span}\{\vec{u}, \vec{v}\}$ where $\vec{u}$ and $\vec{v}$ are LI, subspace has dimension 2
iv) the entire space $\mathbb{R}^3$: $\text{span}\{\vec{u}, \vec{v}, \vec{w}\}$ where $\vec{u}$, $\vec{v}$, and $\vec{w}$ are LI subspace has dimension 3

note: row operations don't change the null space or the row space of a matrix

if $A \xrightarrow{\text{row ops}} B$

generally the RREF

then $\mathrm{Null}(A) = \mathrm{Null}(B)$

and $\mathrm{Row}(A) = \mathrm{Row}(B)$

<u>but</u> row operations do change the column space of a matrix

example: $A = \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix} \xrightarrow[R_2 - 2R_1]{} \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix} = B$

row space: $\mathrm{Row}(A) = \mathrm{span}\left([1 \ \ 5], [2 \ \ 10] \right)$

$= \mathrm{span}\left([1 \ \ 5] \right)$ ← from B

column space: $\mathrm{Col}(A) = \mathrm{span}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \end{bmatrix} \right)$

$= \mathrm{span}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)$

NOT THE SAME

$\mathrm{col}(B) = \mathrm{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \end{bmatrix} \right) = \mathrm{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$