Section 4.3: cont'd

Thursday, November 15, 2018 12:53 PM

from last time, we were finding eigenvalues and eigenvectors for

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

and found $\lambda_1 = 1 + 2i$ with $\vec{x} = t \underbrace{\begin{bmatrix} \frac{1}{2} + \frac{1}{2}i \\ 1 \end{bmatrix}}_{\text{can use this as eigenvector}}$

$\underline{\text{or}}$ $\begin{bmatrix} 1+i \\ 2 \end{bmatrix}$

by RREFing $\left[\begin{array}{cc|c} 2-2i & -2 & 0 \\ 4 & -2-2i & 0 \end{array}\right]$

last time, did $\frac{1}{2-2i} R_1$ to get the first entry to equal one

$\underline{\text{or}}$ you could use method #2:

$$\left[\begin{array}{cc|c} 2-2i & -2 & 0 \\ 4 & -2-2i & 0 \end{array}\right]$$

$\downarrow (2+2i) R_1$ multiply by the complex conjugate

$\big[\, 4-4i^2 \quad -2(2+2i) \;\big|\; 0 \,\big]$

$$\left[\begin{array}{cc|c} 4-4i^2 & -2(2+2i) & 0 \\ 4 & -2-2i & 0 \end{array}\right]$$

$$\Big\downarrow \text{ simplify } R_1$$

$$\left[\begin{array}{cc|c} 8 & -4-4i & 0 \\ 4 & -2-2i & 0 \end{array}\right]$$

and continue as before

notice now that we have only found

$$\lambda_1 = 1+2i \quad \text{has} \quad \vec{x}_1 = \begin{bmatrix} \overset{1+i}{i+1} \\ 2 \end{bmatrix}$$

we need to find $\vec{x}_2$ for $\lambda_2 = 1-2i$, but we can just state that

$$\lambda_2 = 1-2i \quad \text{has} \quad \vec{x}_2 = \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$$

why? augmented matrix for $\lambda_2 = 1-2i$ is

$$\left[\begin{array}{cc|c} 2+2i & -2 & 0 \\ 4 & -2+2i & 0 \end{array}\right]$$

$$\Big\downarrow \text{ RREF}$$

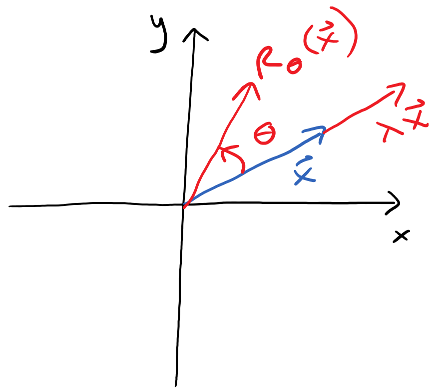
$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2}+\frac{1}{2}i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

complex conj of prev entry in RREF for λ_1

example: explain geometrically why

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

cannot have real eigenvalues if $0 < \theta < 180°$



if R_θ has eigenvalue λ and eigenvector $\vec{x}$, then

$$R_\theta(\vec{x}) = \lambda\vec{x}$$

but from diagram $R_\theta(\vec{x})$ is not in the same direction as $\lambda\vec{x}$

$$R_\theta(\vec{x}) \neq \lambda\vec{x}$$

λ is not a real number

example: find the eigenvalues of $R(60°)$.

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

answer: $$R(60°) = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 1/2 - \lambda & -\sqrt{3}/2 \\ \end{vmatrix} = 0$$

$$\begin{vmatrix} \tfrac{1}{2}-\lambda & -\tfrac{\sqrt{3}}{2} \\ \tfrac{\sqrt{3}}{2} & \tfrac{1}{2}-\lambda \end{vmatrix} = 0$$

$$\left(\tfrac{1}{2}-\lambda\right)^2 + \frac{3}{4} = 0$$

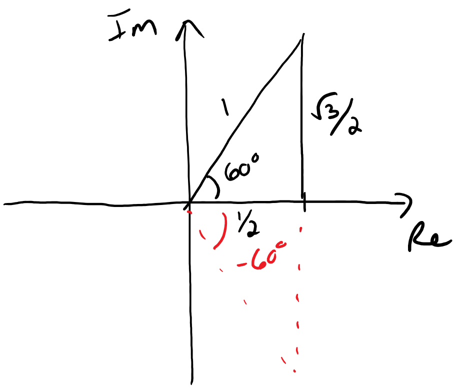
$$\left(\tfrac{1}{2}-\lambda\right)^2 = -\frac{3}{4}$$

$$\tfrac{1}{2}-\lambda = \pm\sqrt{\frac{-3}{4}} = \pm\frac{i\sqrt{3}}{2}$$

$$\frac{1}{2} \pm \frac{i\sqrt{3}}{2} = \lambda$$

$$\lambda = \frac{1 \pm i\sqrt{3}}{2}$$

now write your answer in $re^{i\theta}$ form



$$re^{i\theta} = 1\,e^{i\pi/3}$$

$$\lambda_1 = e^{i\pi/3}$$

$$\lambda_2 = e^{-i\pi/3} \neq e^{i-\pi/3}$$

note: in general, the eigenvalues for R_θ are

$e^{i\theta}$ and $e^{-i\theta}$