Section 5.2: Cont'd

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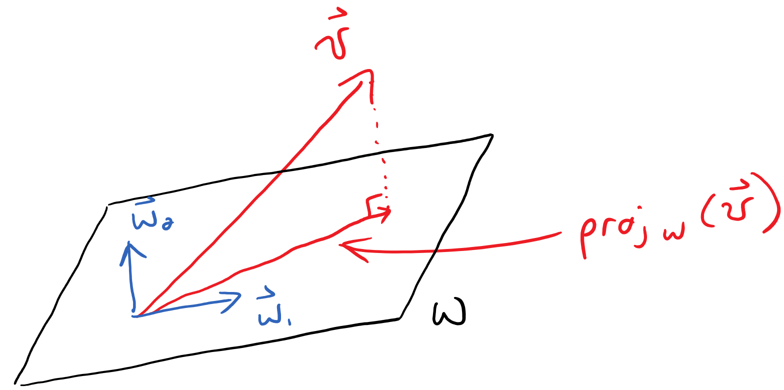
orthogonal projections:

in $\mathbb{R}^3$:



$\vec{w}_1$ and $\vec{w}_2$ are in the plane or W

$W = \text{span}(\vec{w}_1, \vec{w}_2)$

$\vec{w}_1 \perp \vec{w}_2$

so $\text{proj}_W(\vec{v}) = \text{proj}_{\vec{w}_1}(\vec{v}) + \text{proj}_{\vec{w}_2}(\vec{v})$

definition: Let W be a subspace of $\mathbb{R}^N$ and the $\{\vec{w}_1, \vec{w}_2, \vec{w}_3, \dots, \vec{w}_k\}$ be an orthogonal basis of W.

then

$$\begin{cases} \text{proj}_W(\vec{v}) = \text{proj}_{\vec{w}_1}(\vec{v}) + \text{proj}_{\vec{w}_2}(\vec{v}) + \dots \text{proj}_{\vec{w}_k}(\vec{v}) \\ \text{perp}_W(\vec{v}) = \vec{v} - \text{proj}_W(\vec{v}) \end{cases}$$

example: let $W = \text{span}\left(\overset{\vec{w}_1}{\begin{bmatrix} 1 \\ \vdots \end{bmatrix}}, \overset{\vec{w}_2}{\begin{bmatrix} 0 \\ \vdots \end{bmatrix}} \right)$.

example: let $W = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right)$.

Find $\text{proj}_W(\vec{v})$ for $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$.

[note: $\vec{w}_1 \cdot \vec{w}_2 = 0$ so $\vec{w}_1 \perp \vec{w}_2$]

$$\text{proj}_W(\vec{v}) = \text{proj}_{\vec{w}_1}(\vec{v}) + \text{proj}_{\vec{w}_2}(\vec{v})$$

$$= \left(\frac{\vec{v} \cdot \vec{w}_1}{\vec{w}_1 \cdot \vec{w}_1} \right) \vec{w}_1 + \frac{\vec{v} \cdot \vec{w}_2}{\vec{w}_2 \cdot \vec{w}_2} \vec{w}_2$$

$$= \left(\frac{2+1-5}{1+1+1} \right) \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \left(\frac{0+1+5}{1+1} \right) \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= -\frac{2}{3} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2/3 \\ 7/3 \\ 11/3 \end{bmatrix}$$

this is the component of $\vec{v}$ in the plane W

Find $\text{perp}_W(\vec{v})$.

$$\text{perp}_W(\vec{v}) = \vec{v} - \text{proj}_W(\vec{v})$$

$$= \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} - \begin{bmatrix} -2/3 \\ 7/3 \\ 11/3 \end{bmatrix}$$

$$= \begin{bmatrix} 8/3 \\ -4/3 \\ 4/3 \end{bmatrix}$$

quick check: $\text{perp}_W(\vec{v}) \cdot \vec{w}_1 = 0$
$\text{perp}_W(\vec{v}) \cdot \vec{w}_2 = 0$