Section 7.3: cont'd

Wednesday, December 5, 2018 11:39 AM

recall: we were fitting a straight line through the points (0, 3), (1, 3), (1, 6).

we said that $y = mx + b$

plug in points

$$3 = 0m + b$$
$$3 = 1m + b$$
$$6 = 1m + b$$

$$\underbrace{\begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}}_{\vec{b}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b \\ m \end{bmatrix}}_{\vec{x}}$$

$$\vec{x}_{LS} = (A^T A)^{-1} A^T \vec{b}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ 9 \end{bmatrix}$$

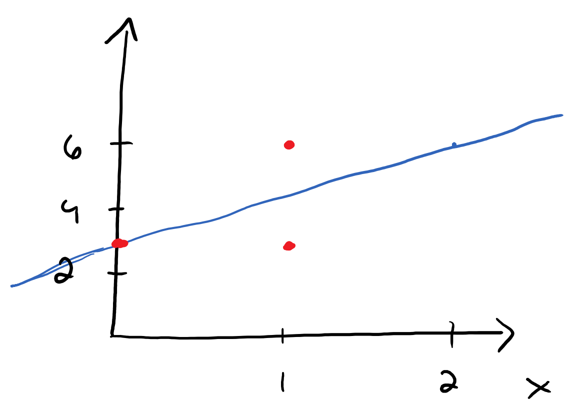
$$\vec{x}_{LS} = (A^TA)^{-1} A^T \vec{b}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 12 \\ 9 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3/2 \end{bmatrix} = \begin{bmatrix} b \\ m \end{bmatrix}$$

y-int = 3
slope = 3/2

least-squares fit: $y = \frac{3}{2}x + 3$



note: (not tested)

if you use $y = a + bx$, you get in general

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

why do we care?

suppose you want to fit $y = a + bx + cx^2$

$$\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$