Review:

Friday, December 7, 2018 1:31 PM

Calculate the determinant of A, where

expand down col 3

$$A = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & 5 & 7 \\ 7 & 9 & 0 & 2 \\ 4 & 0 & 0 & 8 \end{bmatrix} \qquad \begin{bmatrix} + & - & + & - \\ - & + & - & \\ & & + & \\ & & - & \end{bmatrix}$$

$$\det(A) = -5 \begin{vmatrix} 2 & -1 & 3 \\ 7 & 9 & 2 \\ 4 & 0 & 8 \end{vmatrix}$$

$$= -420$$

Consider the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix}$$

which has RREF $= \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

a) find bases for the row space, column space, and null space.

basis for row space $= \{ [1 \;\; 0 \;\; 1 \;\; 0 \;\; -1], \; [0 \;\; 1 \;\; 2 \;\; 0 \;\; 3],$
$[0 \;\; 0 \;\; 0 \;\; 1 \;\; 4] \}$

note: if you were asked for a basis for the row space in terms of the original rows, then you can find the column space of A^T and then transpose those vectors

basis for column space $= \left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

null space: $A\vec{x} = \vec{0}$

$$\begin{aligned} x_1 + x_3 - x_5 &= 0 \\ x_2 + 2x_3 + 3x_5 &= 0 \\ x_4 + 4x_5 &= 0 \end{aligned}$$

let $x_3 = s$
$x_5 = t$

$$\begin{cases} x_1 = -s + t \\ x_2 = -2s - 3t \\ x_3 = s \\ x_4 = -4t \\ x_5 = t \end{cases} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

basis for null space $= \left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right\}$

note: we said before that if the question asked for a basis for row(A) in terms of the original rows, then

$$A^T \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

the column space of A^T is the first three columns so the row space would be the first three rows.

b) give the rank and nullity for this matrix

rank: 3 ← number of leading ones
nullity: 2

c) if $W = \text{row}(A)$, give a basis for $W^\perp$

basis for $W^\perp = \{ [-1 \quad -2 \quad 1 \quad 0 \quad 0],$
$[1 \quad -3 \quad 0 \quad -4 \quad 1] \}$

Are the following vectors linearly independent? Justify your answer.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

method #1: $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 2 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3 = 0$
only has $a = b = c = 0$

Yes

method #2: $a\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

note: this will tell you if third vector is a linear combo of the first two

- you can tell that the second vector isn't parallel to the first by inspection