Section 1.1: The Geometry and Algebra of Vectors

Wednesday, September 07, 2022 12:31 PM

vectors in the plane:

a vector is a directed line segment that corresponds to a displacement from one point A to another point B

in general, points A and B have coordinates

(x_1, y_1) and (x_2, y_2) for 2D vectors

points have round brackets

then vector $\vec{AB}$ can be written as

$$[x_2 - x_1, \; y_2 - y_1]$$

written as a row vector $[3, -2]$

can also write as a column vector $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

note: $\vec{AB}$ is <u>from</u> A <u>to</u> B

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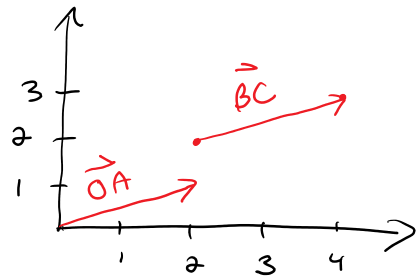
equality of vectors:



these two vectors are <u>equal</u>

vector $\vec{OA}$ is said to be in <u>standard position</u> because tail is at the origin

origin

one convention:

$$\overrightarrow{OA} = \vec{A}$$

O is for origin

notation: in 2D,

$$\vec{0} = [0, 0]$$

the zero vector

← hard to draw! but a perfectly good vector

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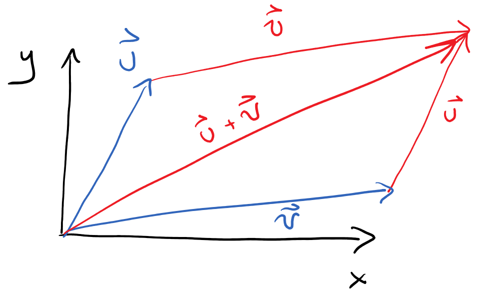
the set of all vectors with two components is written

$\mathbb{R}^2$ (pronounced "R two")

$\mathbb{R}^3$ three components

$\mathbb{R}^n$ n components

vector addition:



$$\vec{u} = [u_1, u_2]$$

$$\vec{v} = [v_1, v_2]$$

$$\vec{u} + \vec{v} = [u_1 + v_1, u_2 + v_2]$$

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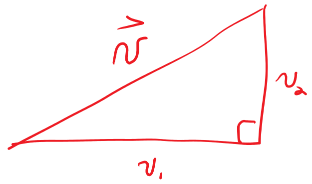
scaling a vector:

$\vec{nr}$

scaling a vector:



$\vec{v}$ is a vector

c is a constant

if $\vec{v} = [v_1, v_2]$, then $c\vec{v} = [cv_1, cv_2]$

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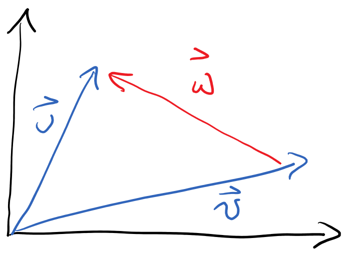
vector subtraction



$\vec{v} + \vec{w} = \vec{u}$

$\vec{w} = \vec{u} - \vec{v}$

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linear combinations of vectors: definition is on handout

example: $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$

coefficients

then $\vec{w}$ is a linear combination of

$\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}0\\2\end{bmatrix}$

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omit Binary vectors and Modular Arithmetic