Section 2.1: Intro to Systems of Linear Equations

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a system of linear equations is a collection of a finite number of linear equations

example:

$$\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ x_1 \quad\quad - x_3 = 8 \\ 3x_1 + x_2 - 6x_3 = 9 \end{cases}$$

"linear" means variables are all to a power of 1

the solution set to a system is the set of all vectors that satisfy all equations simultaneously

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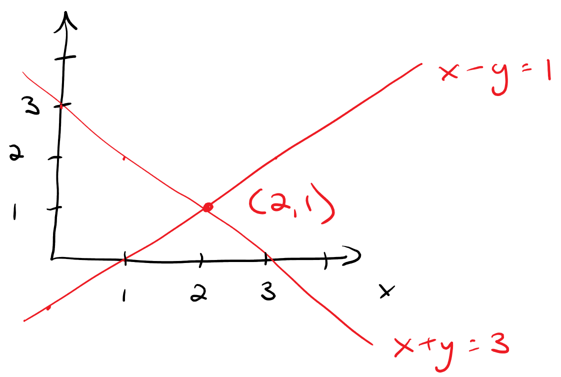
example: consider the system $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$ the only solution

consider the system: $\begin{cases} 3x - y = 6 \\ 6x - 2y = 12 \end{cases}$

→ the same line! infinitely many solutions (all points on line)

how do you write the solution set?

$$3x - y = 6$$
$$y = 3x - 6$$

↑ now sub in parameter t

$$\begin{cases} x = t \\ y = 3t - 6 \end{cases} \quad \text{or} \quad \begin{bmatrix} t \\ 3t-6 \end{bmatrix}$$

solve the system $\begin{cases} 2x - y = 1 \\ 4x - 2y = 3 \end{cases}$

two parallel lines, so system has no solution

in general, for any system of linear equations, there is either

1. a unique solution
2. no solutions
3. infinitely many

two systems of linear equations are equivalent iff (if and only if) they share the same solution set

for any system of equations, the following "elementary operations" do not change the solution:

1. interchange two equations
2. scale any equation by a non-zero constant
3. add a multiple of an equation to another

example: consider the following:

example: consider the following:

$$\begin{cases} x + 2y = 8 & (a) \\ 3x - 4y = -6 & (b) \end{cases}$$

replace (b) by (b) − 3(a)

$$\begin{cases} x + 2y = 8 \\ \quad -10y = -30 \end{cases}$$

solve by back substitution:

$y = 3$

$x + 2(3) = 8$ so $x = 2$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

augmented matrix:

$$\begin{cases} x + 2y = 8 \\ 3x - 4y = -6 \end{cases} \quad \text{rewrite as} \quad \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 3 & -4 & -6 \end{array}\right]$$

variables constants

an augmented matrix is made up of coefficients of variables and, to the right of the vertical bar, the constants

elementary row operations

1. swap row i with row j $\quad R_i \Leftrightarrow R_j$
2. multiply row i by non-zero constant $\quad cR_i$
3. replace row i by itself plus a constant times row j $\quad R_i + cR_j$

example:

$$\begin{cases} x + 2y = 8 \\ 3x - 4y = -6 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 8 \\ 3 & -4 & -6 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & -10 & -30 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{10} R_2} \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

Solution set is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

terminology:

- linear system
  - inconsistent (no solutions)
  - consistent (at least one)
    - exactly one
    - infinitely many