Section 2.4: Applications of Linear Systems

Thursday, October 06, 2022 9:30 AM

handout question #1: given points $(1,-2)$, $(-1,8)$, $(2,-1)$, find $y = ax^2+bx+c$

answer: plug in points:

$(1,-2)$
$$y = ax^2+bx+c$$
$$-2 = a(1)^2 + b(1) + c$$
$$-2 = a + b + c$$

$(-1,8)$
$$8 = a(-1)^2 + b(-1) + c$$
$$8 = a - b + c$$

$(2,-1)$
$$-1 = a(2)^2 + b(2) + c$$
$$-1 = 4a + 2b + c$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 1 & -1 & 1 & 8 \\ 4 & 2 & 1 & -1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$a = 2$
$b = -5$
$c = 1$

$$y = ax^2+bx+c$$
$$\boxed{y = 2x^2 - 5x + 1}$$

handout question #2:

let $x =$ amount of ingredient 1
$y =$ 2
$z =$ 3

magnesium: $10x + 30y + 20z = 120$
vitamin C: $20x + 50y + 30z = 220$
calcium: $60x + 130y + 70z = 620$

$$\left[\begin{array}{ccc|c} 10 & 30 & 20 & 120 \\ 20 & 50 & 30 & 220 \\ 60 & 130 & 70 & 620 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x - z = 6$$
$$y + z = 2$$

$$\begin{cases} x = t + 6 \\ y = 2 - t \\ z = t \end{cases}$$

now, assuming that you can only have integer amounts of the ingredients:

also, cannot have negative amounts

$x \geq 0$	$t+6 \geq 0$	$t \geq -6$
$y \geq 0$	$2-t \geq 0$	$t \leq 2$
$z \geq 0$	$t \geq 0$	$t \geq 0$

if t can be any real number then stop here $\qquad 0 \leq t \leq 2$

but t has to be an integer then:

$t = 0$ then $x = 6$, $y = 2$, $z = 0$

$t = 1$ then $x = 7$, $y = 1$, $z = 1$

$t = 2$ then $x = 6$, $y = 0$, $z = 2$

for integer values 3 possible solutions

---

what if the RREF for this word problem looked like

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

no solution

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 1 \end{array}\right]$$ no solution

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

one solution

5 units of ingredient 1

3 — 2

8 — 3

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

word problem as posed cannot have negative quantities

– no solution

---

handout question #3:

balance $NH_3 + O_2 \rightarrow N_2 + H_2O$

add coeffs: $wNH_3 + xO_2 \rightarrow yN_2 + zH_2O$

atoms:

	LHS = RHS
N :	$w = 2y$
H :	$3w = 2z$
O :	$2x = z$

so
$$\begin{cases} w - 2y = 0 \\ 3w - 2z = 0 \\ 2x - z = 0 \end{cases}$$

Section 2.4 cont'd 2022/10/07

$$\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right] \end{array} \xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right]$$

so
$$w - \tfrac{2}{3}z = 0$$
$$x - \tfrac{1}{2}z = 0$$
$$y - \tfrac{1}{3}z = 0$$

$$\begin{cases} w = \tfrac{2}{3}t \\ x = \tfrac{1}{2}t \\ y = \tfrac{1}{3}t \\ z = t \end{cases}$$

we'll pick $t = 6$ to get the smallest set of positive integers so $w = \frac{2}{3}t = 4$

$x = 3$

$y = 2$

$z = 6$

with $\boxed{4NH_3 + 3O_2 \rightarrow 2N_2 + 6H_2O}$

---

handout question #4 - network analysis

for each node (intersection), traffic in = traffic out

A: $10 + 10 = f_1 + f_2$

B: $f_1 + f_3 = 20 + 5$

C: $15 + 15 = f_3 + f_4$

D: $f_2 + f_4 = 15 + 10$

$$\begin{cases} f_1 + f_2 = 20 \\ f_1 + f_3 = 25 \\ f_3 + f_4 = 30 \\ f_2 + f_4 = 25 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 1 & 0 & 25 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 1 & 0 & 1 & 25 \end{array}\right] \xrightarrow{RREF} \begin{array}{c} \begin{array}{cccc} f_1 & f_2 & f_3 & f_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

so let $f_4 = t$ and then

$$\boxed{\begin{cases} f_1 = t - 5 \\ f_2 = -t + 25 \\ f_3 = -t + 30 \\ f_4 = t \end{cases}}$$

answer to part (a)

b) $f_4 = t = 10$ so

$$\begin{cases} f_1 = 5 \\ f_2 = 15 \\ f_3 = 20 \\ f_4 = 10 \end{cases}$$

c) we want all of f_1, f_2, f_3, and $f_4 \geq 0$

so

$$\begin{aligned} t - 5 &\geq 0 \\ -t + 25 &\geq 0 \\ -t + 30 &\geq 0 \\ t &\geq 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} t &\geq 5 \\ 25 &\geq t \\ \cancel{30} &\cancel{\geq t} \\ \cancel{t} &\cancel{\geq 0} \end{aligned}$$

so $5 \le t \le 25$

when $t = 5$

$$\boxed{\begin{aligned} f_1 &= 0 \\ f_2 &= 20 \\ f_3 &= 25 \\ f_4 &= 5 \end{aligned}}$$

when $t = 25$

$$\boxed{\begin{aligned} f_1 &= 20 \\ f_2 &= 0 \\ f_3 &= 5 \\ f_4 &= 25 \end{aligned}}$$

question #5 from handout:

- find all possible combinations of 20 coins (nickels, dimes, quarters) that will make exactly \$3.00.

answer: $n + d + q = 20$

$5n + 10d + 25q = 300$ (or $0.05n + 0.10d + 0.25q = 3$)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 5 & 10 & 25 & 300 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -20 \\ 0 & 1 & 4 & 40 \end{array}\right]$$

so $n - 3q = -20$
$d + 4q = 40$

$$\begin{cases} n = 3t - 20 \\ d = -4t + 40 \\ q = t \end{cases}$$

BUT want all of n, d, and q to be ≥ 0 and integer

$3t - 20 \ge 0$
$40 - 4t \ge 0$
$t \ge 0$

so $t \ge 6\frac{2}{3}$
$t \le 10$
$t \ge 0$

so $\begin{cases} n = 3t - 20 \\ d = -4t + 40 \\ q = t \end{cases}$ where $7 \le t \le 10$

totally optional and possibly annoying check:

$t = 7$

$n = 1$ $\quad$ $5¢$

$d = 12$ $\quad$ $120¢$

$q = 7$ $\quad$ $175¢$

$$\begin{array}{r} 5¢ \\ 120¢ \\ 175¢ \\ \hline 300¢ \end{array}$$