Section 3.2: Matrix Algebra

Wednesday, October 12, 2022 1:11 PM

see Properties of Matrices handout for "Algebraic Properties"

- basically, matrices add like you'd expect

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definition: $c_1 A_1 + c_2 A_2 + \dots + c_n A_n$ is a <u>linear combination</u> of the matrices $A_1, A_2 \dots A_n$

example: Is $B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ a LC of (linear combo)

$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$?

answer: $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = c_1 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

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section 3.2 cont'd:
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$$\begin{cases} 2 = c_2 + c_3 & \text{top left entry} \\ 1 = c_1 + c_3 & \text{top right} \\ -3 = -c_1 + c_3 & \text{bottom left} \\ 2 = c_2 + c_3 & \text{bottom right} \end{cases}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & -3 \\ 0 & 1 & 1 & 2 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$c_1 = 2$

$c_2 = 3$
$c_3 = -1$

$$B = 2A_1 + 3A_2 - A_3$$

$\boxed{\text{yes}}$

definition: the span of a set of $M \times N$ matrices is the set of all linear combinations of the matrices

so, from our previous example,

B is in $\text{span}(A_1, A_2, A_3)$

definition: matrices $A_1, A_2, A_3 \ldots A_n$ are linearly independent (LI) if

$$c_1 A_1 + c_2 A_2 + \ldots + c_n A_n = 0$$

only has the trivial solution $c_1 = c_2 = c_3 = \ldots c_n = 0$

- otherwise, they are linearly dependent (LD)

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example: Are $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ linearly independent?

answer: $c_1 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

where $c_2 + c_3 = 0$

$$\begin{aligned} c_1 \quad\quad + c_3 &= 0 \\ -c_1 \quad\quad + c_3 &= 0 \\ c_2 + c_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

so the solution

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$

is the unique solution

so A_1, A_2, A_3 are LI

- look at Properties of Matrices handout
  "Properties of Matrix Multiplication" section

warning: recall that in general, $BA \neq AB$
for matrices A and B

$$(A+B)^2 = (A+B)(A+B)$$

$$= A^2 + \underbrace{AB + BA}_{} + B^2 \qquad \text{okay so far}$$

but cannot
assume that $AB = BA$
so this is <u>not</u> $2AB$

look at handout again, "Properties of Transpose"

the one property of the transpose that you really need to know:

$$(AB)^T = B^T A^T$$

if you think about it, this actually makes a certain amount of sense

$$AB = \begin{array}{cc} 2\times 3 & 3\times 1 \\ \begin{bmatrix} 2 & 3 & 7 \\ 4 & 0 & 9 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \end{array} = \begin{array}{c} 2\times 1 \\ \begin{bmatrix} \; \\ \; \end{bmatrix} \end{array}$$

what would $A^T B^T$ look like?

$$A^T B^T = \begin{array}{cc} 3\times 2 & 1\times 3 \\ \begin{bmatrix} 2 & 4 \\ 3 & 0 \\ 7 & 9 \end{bmatrix} & \begin{bmatrix} 1 & 1 & 4 \end{bmatrix} \end{array} \Leftarrow \text{not defined!}$$