Date: Fall 2021

Name: _____

Instructor: Patricia Wrean

Math 251
Test 3

Total = $\frac{\quad}{20}$

Show your work. All of the work on this test must be your own.

$$\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{u}}\,\mathbf{u}$$

GOOD LUCK!

1. (5 points) Consider the following complex numbers.

$$z_1 = 3 - i, z_2 = 5e^{i\pi}, z_3 = 4e^{i2\pi/3}$$

Evaluate the following. You may leave your answer in either rectangular or polar form, your choice. If using polar form, your angles should satisfy $0 \leq \theta < 2\pi$ or $0 \leq \theta < 360^\circ$, as appropriate.

(a) $z_1 + z_2$

(b) $z_2 z_3$

(c) z_3^5

2. (3 points) Use Cramer's Rule to solve the following system of linear equations.

$$\begin{cases} 2x + 7y = -5 \\ -x + hy = 1 \end{cases}$$

Note: your answer will be in terms of h.

3. (6 points) Consider the matrix A given below. Find diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$. Note: you do not need to calculate P^{-1}.

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

4. (6 points) Consider the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{u}$ below and let subspace $W = \text{span}(\mathbf{v}_1, \mathbf{v}_2)$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \mathbf{u} \begin{bmatrix} 3 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

(a) Show that $\{\mathbf{v}_1, \mathbf{v}_2\}$ is an orthogonal set.

(b) Find the associated orthonormal set for $\{\mathbf{v}_1, \mathbf{v}_2\}$.

(c) Find $\text{proj}_W(\mathbf{u})$ and $\text{perp}_W(\mathbf{u})$.