

Math 251 – Test 2

November 2, 2018

Name: _____

Instructor: Patricia Wrean

Total: 25 points

1. (5 points) Find the 2×2 matrix A that satisfies the following.

$$BA - C^T = 3B, \text{ where } B = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 1 \\ 0 & -4 \end{bmatrix}$$

2. (5 points) Solve the system $A\mathbf{x} = \mathbf{b}$ using the following factorization of A .

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 6 \\ 14 \\ -8 \end{bmatrix}$$

3. (5 points) Consider the following matrix.

$$A = \begin{bmatrix} 2 & -2 & 4 \\ -5 & 5 & -10 \\ 2 & 1 & -5 \\ 4 & -7 & 17 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- (a) Find a basis for the row space of A consisting of rows of A .
- (b) Find a basis for the column space of A consisting of columns of A .

4. (5 points) Find the standard matrix A for the following transformations. Note that

$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

- (a) The transformation which reflects a vector in \mathbb{R}^2 about the line $y = -x$.
- (b) The transformation which reflects a vector in \mathbb{R}^2 about the line $y = -x$ and then rotates it counterclockwise by 30° .

5. (5 points) City engineers have been measuring the traffic flow for the streets surrounding a circular park, as shown in the diagram below. Each street is one-way, and the numbers represent the average number of vehicles per minute entering and leaving the intersections during the day.

- (a) Set up and solve a system of linear equations to find the possible flows f_1 , f_2 , and f_3 .
- (b) What are the minimum and maximum possible flows on each street?

