Math 251 – Test 3

November 30, 2018 **Name:** _____
Instructor: Patricia Wrean

Show your work to receive full credit. **Total: 25 points**

1. (5 points) Consider the following complex numbers.

$$z_1 = -2i, z_2 = 3 - 4i, z_3 = 4e^{i\pi/4}$$

Evaluate the following. You may leave your answer in either rectangular or polar form, your choice. Give an exact answer for part (a).

(a) $z_1 + z_3$

(b) $\dfrac{z_1}{z_2}$

2. (5 points) Use Cramer's Rule to solve the following system of linear equations.

$$\begin{cases} x + 2y - z = 2 \\ 3x + 7y - 5z = 5 \\ -x - 2y = 1 \end{cases}$$

3. (3 points) Find all values of x for which the following matrix is not invertible.

$$A = \begin{bmatrix} x & x & 0 \\ x & 3 & 1 \\ 2 & 4 & 3 \end{bmatrix}$$

4. (6 points) Consider the following matrix.

$$A = \begin{bmatrix} -2 & 2 \\ -3 & 5 \end{bmatrix}$$

Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

5. (6 points) Consider $\mathbf{v}$ and subspace W.

$$\mathbf{v} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}, \qquad W = \text{span}\left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \right)$$

   (a) Find an orthogonal basis for W.

   (b) For the basis you found in part (a), express $\mathbf{v}$ as a linear combination of the vectors in that basis.