Date: October 7, 2020

Instructor: Patricia Wrean

Name: Solution Set

Math 251
Test 1

Total = $\frac{\quad}{25}$

- **Show your work. All of the work on this test must be your own.** While writing this test, you may not consult any other person, website, or other resource not listed below. If you have a question during the test, you may email me.
- Here is a list of the resources that you are allowed to use during this test:
  - your own notes
  - lecture notes, videos, handouts, practice questions, and practice tests from either my website at http://wrean.ca/math251 or the Math 251 websites of any of the other instructors linked on the landing page of my site
  - your textbook (Poole), or any of the texts listed on the Textbook page at http://wrean.ca/math251/math251_textbook.htm
  - the Math 251 D2L website
  - the Math 251 WeBWorK online homework site
  - a scientific calculator. You may not use a calculator with graphing capability. If you like, you may use a scientific calculator app like the one at Desmos: https://www.desmos.com/scientific
  - if you have questions during the test, you may email me

in Fall 2020, this course was fully online with online tests

in Fall 2021 we will have in-person test with different rules

- To submit this test, please use the Dropbox feature in the Assignments tab of D2L. Please assemble your answers into a single PDF or Word document, unless you've made other arrangements with me beforehand. Helpful software:
  - Genius Scan app at https://www.thegrizzlylabs.com/genius-scan/
  - CombinePDF at https://combinepdf.com/

GOOD LUCK!

1. (5 points) Consider the two points $P = (2, 0, 1)$ and $Q = (1, 2, 2)$.

2

(a) Write the equation of the line joining these two points. Write your answer in parametric form.

vector to point on line $\vec{p} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

direction vector $\vec{d} = \vec{PQ} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

($\vec{QP}$ also fine, as is any non-zero multiple of $\vec{PQ}$)

If X is a point on the line,

$$\vec{PX} = t\vec{d} = t\vec{PQ}$$

$$\begin{bmatrix} x-2 \\ y-0 \\ z-1 \end{bmatrix} = t\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

only gave vector eqn (-1)

$$\boxed{\begin{cases} x = -t+2 \\ y = 2t \\ z = t+1 \end{cases}}$$

each minor error $-\frac{1}{2}$

3

(b) Find the angle $0 \le \theta \le 180°$ between the vector $\vec{QP}$ and the positive z-axis. Round your answer to the nearest degree.

$\vec{QP} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$ $\hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\|\vec{QP}\| = \sqrt{(1)^2 + (-2)^2 + (-1)^2} = \sqrt{6}$

$\|\hat{k}\| = 1$

incorrect quadrant (-1) $\theta = 65.9052°$ ($= 1.15026$ rads) $= 66°$

rads correct but incorrect conversion to degrees $-\frac{1}{2}$

didn't round as per directions $-\frac{1}{2}$

so $\vec{QP} \cdot \hat{k} = \|\vec{QP}\| \, \|\hat{k}\| \cos\theta$

correct setup but couldn't figure out z direction (-2)

$$\cos\theta = \frac{\vec{QP} \cdot \hat{k}}{\|\vec{QP}\| \, \|\hat{k}\|} = \frac{-1}{\sqrt{6} \cdot 1}$$

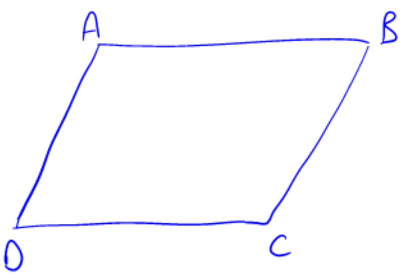
$$\theta = 114.095° \quad (= 1.99133 \text{ rads})$$

$$= 114°$$

2. (6 points) The parallelogram $ABCD$ has vertices at $A = (1, 4, -3)$, $B = (-3, 1, -2)$, and $C = (-1, 5, 0)$. The convention is that the points are named in order as you go around the perimeter of the geometric figure. So point A is connected to points B and D.

3

(a) Is this parallelogram a rhombus? Explain briefly. (A rhombus is a parallelogram whose sides all have the same length.)



$\vec{AB} = \begin{bmatrix} -4 \\ -3 \\ 1 \end{bmatrix}$ $\quad \vec{BC} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ 1

$\|\vec{AB}\| = \sqrt{(-4)^2 + (-3)^2 + 1^2} = \sqrt{26}$ 1

$\|\vec{BC}\| = \sqrt{2^2 + 4^2 + 2^2} = \sqrt{24}$ $(= 2\sqrt{6}$ if you like$)$

sides not same length, so $\boxed{\text{No}}$ 1

3

(b) Calculate the area of this parallelogram.

$$\vec{AB} \times \vec{BC} = \left| \begin{array}{ccc|cc} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ -4 & -3 & 1 & -4 & -3 \\ 2 & 4 & 2 & 2 & 4 \end{array} \right.$$

$$= -6\hat{\imath} + 2\hat{\jmath} - 16\hat{k} - (-6\hat{k} + 4\hat{\imath} - 8\hat{\jmath})$$

$$= -10\hat{\imath} + 10\hat{\jmath} - 10\hat{k}$$

$$\|\vec{AB} \times \vec{BC}\| = \sqrt{-10^2 + 10^2 + (-10)^2} = \sqrt{300}$$

$\boxed{= 3\sqrt{10}}$

either

≈ 17.32051

$\boxed{\approx 17.3}$

3. (5 points) Use Gauss-Jordan elimination to find all solutions of the following linear system. Write your answer in column vector form.

$$\begin{cases} x_1 & +3x_2 & -x_3 & -9x_4 & = & 7 \\ 3x_1 & +9x_2 & +x_3 & -11x_4 & = & 5 \\ -2x_1 & -6x_2 & +1x_3 & +14x_4 & = & -10 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & -9 & 7 \\ 3 & 9 & 1 & -11 & 5 \\ -2 & -6 & 1 & 14 & -10 \end{array}\right] \to \begin{array}{c} \\ R_2-3R_1 \\ R_3+2R_1 \end{array} \left[\begin{array}{cccc|c} 1 & 3 & -1 & -9 & 7 \\ 0 & 0 & 4 & 16 & -16 \\ 0 & 0 & -1 & -4 & 4 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} \\ \frac{1}{4}R_2 \\ R_3-\frac{1}{4}R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 3 & -1 & -9 & 7 \\ 0 & 0 & 1 & 4 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} R_1+R_2 \\ \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 3 & 0 & -5 & 3 \\ 0 & 0 & 1 & 4 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

3

x_1 and x_3 are leading variables

x_2 and x_4 are free variables

so $x_1 + 3x_2 - 5x_4 = 3$

$x_3 + 4x_4 = -4$

now let $x_2 = s$

$x_4 = t$

1

then
$$\begin{cases} x_1 = -3s + 5t + 3 \\ x_2 = s \\ x_3 = -4t - 4 \\ x_4 = t \end{cases}$$

1

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 5 \\ 0 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ -4 \\ 0 \end{bmatrix}}$$

4. (4 points) Find all unit vectors that are perpendicular to the following plane.

$$\begin{cases} x = 7 - 2s \\ y = 3s + 2t \\ z = -1 - 5s - 3t \end{cases}$$

5. (5 points) Consider the line through point P with direction vector $\mathbf{d}$, where P and $\mathbf{d}$ are given below.

$$P = (3, 0, -1), \qquad \mathbf{d} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Calculate the distance from this line to the point $Q = (-1, 2, 3)$.