

Date: November 4, 2020

Name: _____

Instructor: Patricia Wrean

Math 251 Test 2

Total = $\overline{25}$

- **Show your work.** All of the work on this test must be your own. While writing this test, you may not consult any other person, website, or other resource not listed below. If you have a question during the test, you may email me.
- Here is a list of the resources that you are allowed to use during this test:
 - your own notes
 - lecture notes, videos, handouts, practice questions, and practice tests from either my website at <http://wrean.ca/math251> or the Math 251 websites of any of the other instructors linked on the landing page of my site
 - your textbook (Poole), or any of the texts listed on the Textbook page at http://wrean.ca/math251/math251_textbook.htm
 - the Math 251 D2L website
 - the Math 251 WeBWorK online homework site
 - a scientific calculator. You may not use a calculator with graphing capability. If you like, you may use a scientific calculator app like the one at Desmos: <https://www.desmos.com/scientific>
 - the RREF calculator at <https://adrianstoll.com/linear-algebra/row-reduction.html>
 - if you have questions during the test, you may email me
- To submit this test, please use the Dropbox feature in the Assignments tab of D2L. Please assemble your answers into a single PDF or Word document, unless you've made other arrangements with me beforehand. Helpful software:
 - Genius Scan app at <https://www.thegrizzlylabs.com/genius-scan/>
 - CombinePDF at <https://combinepdf.com/>

GOOD LUCK!

1. (4 points) Consider the matrices $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$.

Evaluate the following, if they exist. If they do not exist, say so.

- (a) AC
- (b) CA
- (c) $C^T + C$
- (d) C^{-1}

2. (2 points) Solve the given matrix equation for X and simplify. Assume that all matrices are invertible.

$$AX = B^T A + A$$

3. (4 points) Find an LU factorization of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

4. (4 points) Consider the following 3×4 matrix.

$$A = \begin{bmatrix} 1 & -1 & 5 & -9 \\ 3 & 1 & 3 & -11 \\ -2 & 3 & -13 & 22 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$A^T = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 1 & 3 \\ 5 & 3 & -13 \\ -9 & -11 & 22 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -\frac{11}{4} \\ 0 & 1 & \frac{1}{4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Give the values of $\text{rank}(A)$ and $\text{nullity}(A)$.

(b) Find a basis for $\text{null}(A)$.

5. (5 points) Consider the set of all parabolas $y = ax^2 + bx + c$ that pass through the following two points.

$$(1, 12), (2, -18)$$

Set up and solve a system to find the possible values of a , b , and c . Give your answer in parametric form.

6. (6 points) Consider the following four vectors in \mathbb{R}^3 :

$$\mathbf{u} = \begin{bmatrix} -10 \\ -9 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

where the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ gives

$$T(\mathbf{v}_1) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad T(\mathbf{v}_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad T(\mathbf{v}_3) = \begin{bmatrix} -1 \\ 7 \end{bmatrix}.$$

(a) Express \mathbf{u} as a linear combination of \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 .

(b) Are \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 linearly dependent or linearly independent? Explain briefly.

(c) Find $T(\mathbf{u})$ for the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$.