

Date: December 2, 2020

Name: _____

Instructor: Patricia Wrean

Math 251 Test 3

Total = $\overline{25}$

- **Show your work. All of the work on this test must be your own.** While writing this test, you may not consult any other person, website, or other resource not listed below. If you have a question during the test, you may email me.
- Here is a list of the resources that you are allowed to use during this test:
 - your own notes
 - lecture notes, videos, handouts, practice questions, and practice tests from either my website at <http://wrean.ca/math251> or the Math 251 websites of any of the other instructors linked on the landing page of my site
 - your textbook (Poole), or any of the texts listed on the Textbook page at http://wrean.ca/math251/math251_textbook.htm
 - the Math 251 D2L website
 - the Math 251 WeBWorK online homework site
 - a scientific calculator. You may not use a calculator with graphing capability. If you like, you may use a scientific calculator app like the one at Desmos: <https://www.desmos.com/scientific>
 - the RREF calculator at <https://adrianstoll.com/linear-algebra/row-reduction.html>
 - if you have questions during the test, you may email me
- To submit this test, please use the Dropbox feature in the Assignments tab of D2L. Please assemble your answers into a single PDF or Word document, unless you've made other arrangements with me beforehand. Helpful software:
 - Genius Scan app at <https://www.thegrizzlylabs.com/genius-scan/>
 - CombinePDF at <https://combinepdf.com/>

GOOD LUCK!

1. (3 points) The determinant of matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is equal to 5.

Calculate the determinants of the following matrices.

$$(a) B = \begin{bmatrix} a & b & c \\ 4d & 4e & 4f \\ -g & -h & -i \end{bmatrix}$$

$$(b) C = \begin{bmatrix} g & h & i \\ d & e & f \\ a & b & c \end{bmatrix}$$

$$(c) D = \begin{bmatrix} a + 5d & b + 5e & c + 5f \\ d & e & f \\ g & h & i \end{bmatrix}$$

2. (6 points) Find all of the eigenvalues of the matrix A over the complex numbers. Give bases for each of the corresponding eigenspaces.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

3. (6 points) Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) State the algebraic and geometric multiplicity of each eigenvalue by filling in the following table. Show any necessary work below.

eigenvalue	algebraic multiplicity	geometric multiplicity

(b) Is the matrix diagonalizable? Explain briefly.

4. (4 points) For each of the following matrices, state whether it is orthogonal. If it is, find the associated inverse. If it isn't, give a reason for why it's not.

(a)
$$\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

(b)
$$\begin{bmatrix} -4 & -3 \\ -3 & 4 \end{bmatrix}$$

(c)
$$\begin{bmatrix} \frac{5}{13} & 0 & -\frac{12}{13} \\ \frac{12}{13} & 0 & \frac{5}{13} \\ 0 & 1 & 0 \end{bmatrix}$$

5. (6 points) Consider the subspace spanned by the following set of vectors. Use the Gram-Schmidt process to obtain an orthogonal basis for this subspace.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Note: You do not need to normalize your answers.