Date: Fall 2022

Name: Solution Set

Instructor: Patricia Wrean

# Math 251
# Test 1

**Total =** $\overline{\phantom{00}}$ 20

Show your work. All of the work on this test must be your own.

## GOOD LUCK!

1. (6 points) Consider the following vectors.

$$\mathbf{u} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(a) Calculate the angle $0 \leq \theta < 180°$ between $\mathbf{u}$ and $\mathbf{v}$.

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

$\|\vec{u}\| = \sqrt{1+16+4} = \sqrt{21}$

$\|\vec{v}\| = \sqrt{4+9} = \sqrt{13}$

2

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{-2+0+6}{\sqrt{21}\sqrt{13}}$

$\theta = \arccos\left(\dfrac{4}{\sqrt{21}\sqrt{13}}\right)$ $\boxed{\approx 75.99° \text{ or } 76° \quad \approx 1.33 \text{ rads}}$

(b) Find all unit vectors that are parallel to $\mathbf{u}$.

$\|\vec{u}\| = \sqrt{21}$ from before

no ± −1

2

$\hat{u} = \pm \dfrac{1}{\sqrt{21}} \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$

(c) Compute $||2\mathbf{v} - \mathbf{w}||$.

$2\vec{v} - \vec{w} = 2\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$

2

$\|2\vec{v} - \vec{w}\| = \sqrt{3^2 + 1^2 + 5^2}$

$\boxed{= \sqrt{35} \quad \approx 5.92}$

if did $2\|\vec{v}\| - \|\vec{w}\|$, will get $2\sqrt{13} - \sqrt{3} \approx 5.479$

−1

2. (4 points) Consider the plane $3x + 2y - z = 5$.

(a) Is point $P = (2, -7, -13)$ in this plane? Explain your reasoning.

1

$3(2) + 2(-7) - (-13) = 5$

$6 - 14 + 13 = 5$

$5 = 5$

$\boxed{\text{Yes}}$

(b) Give parametric equations for the line perpendicular to this plane that goes through the point $Q = (-1, 1, 4)$.

direction vector of line $\vec{d} = \vec{N} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

3

so $\vec{x} = \vec{Q} + t\vec{d}$

$= \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

$$\boxed{\begin{cases} x = -1 + 3t \\ y = 1 + 2t \\ z = 4 - t \end{cases}}$$

3. (5 points) Consider three points

$$P = (2, 0, -1), \quad Q = (-1, 3, -2), \quad R = (0, 4, -1).$$

(a) Calculate the area of the triangle $PQR$.

$\vec{PQ} = \begin{bmatrix} -3 \\ 3 \\ -1 \end{bmatrix}$ $\vec{PR} = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & -1 \\ -2 & 4 & 0 \end{vmatrix}$$

2

$$= 2\hat{j} - 12\hat{k} + 4\hat{i} + 6\hat{k} = 4\hat{i} + 2\hat{j} - 6\hat{k}$$

area $\Delta PQR = \frac{1}{2}\|\vec{PQ} \times \vec{PR}\| = \frac{1}{2}\sqrt{16+4+36} = \boxed{\frac{1}{2}\sqrt{56} \approx 3.74}$

(b) Give the general equation for the plane that contains points $P$, $Q$, and $R$.

given cross product above, normal is $\vec{N} = \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}$

$$Ax + By + Cz = D$$
$$4(2) + 0 - 6(-1) = D$$
$$D = 14$$

1

$$\boxed{\begin{array}{l} 4x + 2y - 6z = 14 \\ \text{or } 2x + y - 3z = 7 \end{array}}$$

(c) Is the vector $\mathbf{u} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ parallel to the plane you found in part (b)? Explain briefly.

If $\vec{u}$ is parallel to the plane, then $\vec{u} \perp \vec{N}$

and $\vec{u} \cdot \vec{N} = 0$

2

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix} = 1(4) + 4(2) + 2(-6)$$

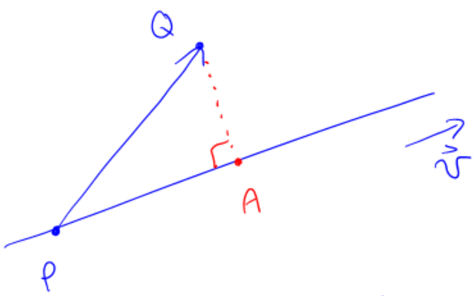
$$= 4 + 8 - 12$$
$$= 0$$

$\boxed{\text{Yes}}$

4. (5 points) Consider the line that goes through the point $P$ and has direction vector $\mathbf{v}$ where

$$P = (1, -1, 1) \qquad \mathbf{v} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$$

Find the point on this line that is closest to point $Q = (4, -2, 1)$




$$PQ = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

1

$$\vec{PA} = \operatorname{proj}_{\vec{v}}(\vec{PQ})$$

$$= \frac{\vec{v} \cdot \vec{PQ}}{\vec{v} \cdot \vec{v}} \vec{v}$$

2

$$= \frac{3+3}{1+9+4} \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = \frac{6}{14} \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = \frac{3}{7} \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$$

$$\vec{PA} = \vec{A} - \vec{P}$$

$$A = \vec{P} + \vec{PA}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3/7 \\ -9/7 \\ 6/7 \end{bmatrix} = \begin{bmatrix} 10/7 \\ -16/7 \\ 13/7 \end{bmatrix}$$

2

$$\boxed{A = \left(10/7, -16/7, 13/7\right)}$$