Math 251
Test 2

Total = $\overline{20}$

Show your work. All of the work on this test must be your own.

GOOD LUCK!

1. (4 points) Consider the matrix $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$.

    Evaluate the following, if they exist. If they do not exist, say so.

    (a) AA^T

    (b) $A + A^T$

    (c) $A^T A$

    (d) A^2

2. (2 points) Solve the given matrix equation for X. Assume that all matrices are invertible.

$$(BX)^{-1} = B^{-1}A$$

3. (4 points) Find an LU factorization of the following matrix.

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{bmatrix}$$

4. (5 points) Consider the following matrix A.

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 2 & 1 & 6 & 1 & 3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Give the values of $\text{Rank}(A)$ and $\text{Nullity}(A)$.

(b) Find a basis for $\text{Null}(A)$.

5. (5 points) Find the equation of the parabola that passes through the points $(1, -2)$, $(-2, 10)$, and the origin.

Set up and solve a system to find this parabola, writing your answer in the form $y = ax^2 + bx + c$. Use Gauss-Jordan elimination and be sure to specify which row operations you are using.