Date: Fall 2022

Name: ______________________

Instructor: Patricia Wrean

Math 251
Test 3

Total = $\overline{\quad}$ 20

Show your work. All of the work on this test must be your own.

$$\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{u}}\,\mathbf{u}$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

GOOD LUCK!

1. (5 points) Consider the transformation T which rotates a vector in $\mathbb{R}^2$ clockwise by 135° and then reflects it about the line $y = -x$.

   (a) Find the standard matrix A for this transformation.

   (b) Now evaluate $T(\mathbf{v})$ for $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

2. (4 points) Find all values of z such that $z^6 = -64i$.

   Give exact answers. You may leave your answer in either rectangular or polar form, your choice. If using polar form, your angles should satisfy $0 \leq \theta < 2\pi$ or $0 \leq \theta < 360°$, as appropriate.

3. (2 points) Compute the determinant of the following matrix.

$$A = \begin{bmatrix} 2 & 0 & 3 & 5 \\ -1 & -2 & 0 & 3 \\ 0 & 2 & 0 & 0 \\ 3 & -3 & k & 0 \end{bmatrix}$$

Your answer should be in terms of k.

4. (5 points) Consider the matrix A given below.

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

(a) Find diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$.

(b) Using your result for part (a), calculate A^6.

5. (4 points) Consider the set of vectors $\mathcal{B}$ below and also the vector $\mathbf{u}$. $\mathcal{B}$ is a basis of $\mathbb{R}^3$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\} \quad \text{and} \quad \mathbf{u} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}$$

(a) Show that $\mathcal{B}$ is an **orthogonal** basis of $\mathbb{R}^3$.

(b) Using the fact that $\mathcal{B}$ is an **orthogonal** basis, express the vector $\mathbf{u}$ as a linear combination of the vectors in $\mathcal{B}$.