Math 252: Name that 1st-order DE!

Name the type of differential equation. Types include:

- separable
- linear 1st order
- exact
- homogeneous
- Bernoulli
- reducible with $u = f(Ax + By + C)$

When appropriate, also specify the first step or steps you would take to solve the DE, such as giving the substitution or integrating factor. (Don't bother to actually solve them.)

1. $(1+x)\,dy - y\,dx = 0$
2. $\dfrac{dx}{dt} = x\left(3x^3t - 1\right)$
3. $\cos x\,dy + y\sin x\,dx = dx$
4. $x^3\dfrac{dy}{dx} = y^2 - 4$
5. $\left(x^2 + y^2\right)dy - \left(x^2 - xy\right)dx = 0$
6. $\dfrac{dy}{dx} = \tan(x - y)$
7. $x\dfrac{dy}{dx} - 4y = x^6e^{-x}$
8. $\left(e^{2y} - 1\right)dx + \left(2xe^{2y} + y\right)dy = 0$
9. $\dfrac{dN}{dt} + N = Nt\,e^{t+2}$
10. $\sin x\,dx + y^3\cos x\,dy = 0$
11. $\left(x^2 + 9\right)\dfrac{dy}{dx} + xy = 5\left(x^2 + 9\right)$
12. $(x + y)^2\,dx + \left(2xy + x^2 - 1\right)dy = 0$
13. $\dfrac{dy}{dx} = -\dfrac{x^3}{x^2y + y^3}$
14. $x\,dy + y\,dx = x^2y^2dx$
15. $\dfrac{dy}{dx} = xy - 3y + x - 3$
16. $\sqrt{r}\,\dfrac{dr}{dt} - \sqrt{r^3} = 4r^2t$

Answers:

1. Rewrite to get $\frac{dy}{y} = \frac{dx}{1+x}$, separable.

2. Rewrite to get $\frac{dx}{dt} + x = 3x^4 t$, Bernoulli with n = 4, so substitute with $u = y^{1-n} = y^{-3}$.

3. Rewrite to get $\frac{dy}{dx} + y\tan x = \sec x$, linear with integrating factor $e^{\int P(x)dx} = e^{\int \tan x\, dx}$.

4. Rewrite to get $\frac{dy}{y^2 - 4} = \frac{dx}{x^3}$, separable.

5. Rewrite to get $\frac{dy}{dx} = \frac{x^2 - xy}{x^2 + y^2}$, homogeneous. So substitute $y = ux$.

6. Reduce to separable with $u = x - y$, and $\frac{du}{dx} = 1 - \frac{dy}{dx}$.

7. Rewrite to get $\frac{dy}{dx} - \frac{4}{x} y = x^5 e^{-x}$, linear with integrating factor $e^{\int P(x)dx} = e^{\int -\frac{4}{x}dx}$

8. Exact, since $\frac{\partial}{\partial y}\left(e^{2y} - 1\right) = \frac{\partial}{\partial x}\left(2xe^{2y} + y\right)$. Alternatively, it's linear if you rewrite it as $\frac{dx}{dy} + \frac{2e^{2y}}{e^{2y} - 1} x = -\frac{y}{e^{2y} - 1}$

9. Rewrite to get $\frac{dN}{N} = \left(t\, e^{t+2} - 1\right) dt$, separable. (Or linear, if you insist.)

10. Rewrite to get $\tan x\, dx + y^3\, dy = 0$, separable.

11. Rewrite to get $\frac{dy}{dx} + \frac{x}{x^2 + 9} y = 5$, linear with integrating factor $e^{\int P(x)dx} = e^{\int \frac{x}{x^2+9}dx}$

12. Exact, since $\frac{\partial}{\partial y}(x+y)^2 = \frac{\partial}{\partial x}\left(2xy + x^2 - 1\right)$.

13. Homogeneous, and since M is simpler than N, substitute $x = uy$.

14. Rewrite to get $\frac{dy}{dx} + \frac{1}{x} y = xy^2$, Bernoulli with n = 2, so $u = y^{1-n} = y^{-1}$.

15. Rewrite to get $\frac{dy}{dx} = (x-3)(y+1)$, separable.

16. Rewrite to get $\frac{dr}{dt} - r = 4r^{3/2} t$, Bernoulli with n = 3/2, so $u = r^{1-n} = r^{-1/2}$.