Explainer: Integrating Factor for Linear 1st order DEs

Wednesday, October 30, 2019 3:03 PM

Consider the term $y\, e^{\int P(x)\,dx}$.

What is $\frac{d}{dx}\left(y\, e^{\int P(x)\,dx} \right)$?

answer:

$$\frac{d}{dx}\left(y\, e^{\int P(x)dx} \right) = \frac{dy}{dx}\, e^{\int P(x)dx} + y \frac{d}{dx}\, e^{\int P(x)dx}$$

$$= \frac{dy}{dx}\, e^{\int P(x)dx} + y\, e^{\int P(x)dx} \frac{d}{dx}\left(\int P(x)dx \right)$$

$$= \frac{dy}{dx}\, e^{\int P(x)dx} + y\, e^{\int P(x)dx} P(x)$$

$$= e^{\int P(x)dx} \left(\frac{dy}{dx} + P(x)\, y \right)$$

why do we care?

$$\frac{dy}{dx} + P(x)\, y = f(x)$$

$$e^{\int P(x)dx} \left(\frac{dy}{dx} + P(x)\, y \right) = f(x)\, e^{\int P(x)dx}$$

$$\frac{d}{dx}\left(y\, e^{\int P(x)dx} \right) = f(x)\, e^{\int P(x)dx}$$

integrate both sides:

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$$\int \frac{d}{dx}\left(y\, e^{\int P(x)\,dx}\right) dx = \int f(x)\, e^{\int P(x)\,dx}\, dx$$

$$y\, e^{\int P(x)\,dx} = \int f(x)\, e^{\int P(x)\,dx}\, dx$$

awkward sum on left becomes a single term

so now just need to compute this