Explainer: Exact Equation IFs

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suppose we have

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

but we perform the check and this DE is not exact.

Can we make it exact? Under certain conditions, yes!

But what are the conditions?

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suppose there is an IF $= \mu(x,y)$. Our DE becomes

$$\mu M\,dx + \mu N\,dy = 0$$

and if $\dfrac{\partial}{\partial y}(\mu M) = \dfrac{\partial}{\partial x}(\mu N)$, then DE is exact.

product rule says:

$$\frac{\partial \mu}{\partial y}\cdot M + \mu\frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x}\cdot N + \mu\frac{\partial N}{\partial x}$$

now what? let's suppose that μ is only a function of one of the two variables:

if $\mu = \mu(x)$, then $\dfrac{\partial \mu}{\partial y} = 0$

DE is:

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$$\mu \frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x} N + \mu \frac{\partial N}{\partial x}$$

now solve for $\frac{\partial \mu}{\partial x}$:

$$\frac{\partial \mu}{\partial x} = \frac{\mu}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$= \mu \left(\frac{M_y - N_x}{N} \right)$$

function of x only

iff this is depends only on x, can separate and integrate

no y's left, only x, so regular derivative

$$\frac{d\mu}{dx} = \mu \left(\frac{M_y - N_x}{N} \right)$$

$$\frac{d\mu}{\mu} = dx \left(\frac{M_y - N_x}{N} \right)$$

$$IF = \mu(x) = e^{\int \left(\frac{M_y - N_x}{N} \right) dx}$$

so multiply both sides of DE by μ and DE is now exact.

similarly, if $\frac{N_x - M_y}{N}$ is only function of y,

use

$$IF = \mu(y) = e^{\int \frac{N_x - M_y}{M} dy}$$

$IF = \mu(y) = e$