Explainer: Quick Explanation for Constant Coeffs and Cauchy Euler

Tuesday, February 18, 2020 10:00 AM

Constant Coeffs:

$$y'' + by' + cy = f(x)$$

if we assume solutions of form $y = e^{mx}$

then $y' = m e^{mx}$

$y'' = m^2 e^{mx}$

then the homogeneous DE is

$$m^2 e^{mx} + b m e^{mx} + c e^{mx} = 0$$

Factoring out e^{mx} gives

$$e^{mx}(m^2 + bm + c) = 0$$

never zero

So find where $m^2 + bm + C = 0$

by solving for m.

Cauchy-Euler:

$$x^2 y'' + bxy' + cy = g(x)$$

If we assume solutions of the form $y = x^m$

then $y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

The homogeneous DE is therefore

$$x^2 m(m-1)x^{m-2} + bx\, m x^{m-1} + c x^m = 0$$

$$m(m-1) x^m + bm x^m + cx^m = 0$$

Factoring out x^m gives

$$x^m \left[m(m-1) + bm + c \right] = 0$$

$\uparrow$ never zero from interval of solution

So find where $m(m-1) + bm + c = 0$

by solving for m.

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note: this begs the question of what the repeated roots case looks like, but you can use reduction of order to find second solution once you have the first