

Decision Tree for Solving Differential Equations

Tuesday, April 11, 2023 12:05 PM

Solve some DE

is it a system? Yes

will look like

$$\begin{cases} \frac{dx}{dt} = 5x - 2y \\ \frac{dy}{dt} = 3x \end{cases}$$

or $\vec{X}' = \begin{bmatrix} 5 & -2 \\ 3 & 0 \end{bmatrix} X$

use techniques of chapter 8

No

is it Laplace? Yes

it says "solve using Laplace" or contains one of

- piecewise $f(t) = \begin{cases} 0 & t < 3 \\ 1 & t \geq 3 \end{cases}$
- $u(t-a)$ Heaviside
- $\delta(t-a)$ Dirac delta

No

is it a power series question? Yes

it says "find two power series solutions"

$$y = C_0 y_1 + C_1 y_2$$

$$\begin{matrix} y_1 = \\ y_2 = \end{matrix}$$

it says "find first n non-zero terms in series solution to..."

$$y = 5 + 3x^2 - 2x^3 + \frac{1}{4}x^5 + \dots \quad \text{IVP}$$

No

is it second order or Yes

$$ay'' + by' + cy = f(x)$$

is it second order or higher?

Yes

$$ay'' + by' + cy = f(x)$$

constant coeff

$$ax^2y'' + bxy' + cy = f(x)$$

Cauchy-Euler

are you given y_1 ?
(first solution)
reduction of order

give y for homogeneous case or y_c if non-homogeneous

No

but what to do about $f(x)$, the RHS, after you've found y_c ?

if $ay'' + by' + cy = f(x)$
constant coeff

and $f(x)$ contains only products of polynomials ($3x^2 + 2x$) and exponentials and sines/cosines

→ method of undetermined coeffs (watch for "bad case")

if const coeffs

and $f(x)$ has $\ln x$
 $\tan x, \sec x$
 \sqrt{x}
 $\frac{1}{x}$

variation of parameters

if $ax^2y'' + bxy' + cy = f(x)$
Cauchy Euler

is the DE first order?

yes

separable
 $f(x)dx = g(y)dy$

explicit solution - solve for dependent variable
if in x, y , then solve for y

implicit solution - don't bother to solve for y

linear 1st order
 $\frac{dy}{dx} + P(x)y = f(x)$
integrating factor is
 $IF = e^{\int P(x)dx}$

substitution

Bernoulli

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$



$$\frac{dy}{dx} = f(Ax + By + C)$$

$y' = \sin(3x + 2y)$ $\frac{dy}{dx} = \sqrt{5y - 3x}$

homogeneous

homogeneous

$$(x^3 + x^2y)dy + (x^3 + xy^2 + y^3)dx = 0$$

↑ ↑ ↑ ↑ ↑

polynomial
of degree
3

exact

$$M(x,y)dx + N(x,y)dy = 0$$

$$\text{has } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

partial derivatives

- if not exact, can make it exact with IF

is it a word problem?

1st order

proportionality

$$\frac{dy}{dt} = ky$$

examples: population growth

$$\frac{dP}{dt} = kP$$

k will be positive

exponential decay

$$\frac{dP}{dt} = -kP$$

k will be negative

[or just do $\frac{dP}{dt} = -kP$
with k positive]

Newton's Law of Cooling

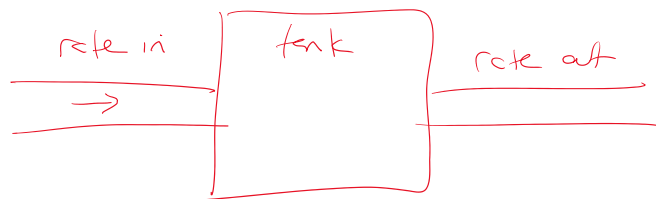
$$\frac{dT}{dt} = k(T - T_{\text{room}})$$

air resistance

$$\vec{F}_{\text{drag}} = k \vec{v}$$

↑
object's speed

mixture problems



$$\frac{dm}{dt} = \text{rate in} - \text{rate out}$$

↑

rate of
change of
mass of salt
or other substance
in tank

2nd order word problems

mass/spring system

external
force
↓

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_{\text{ext}}(t)$$

↑
m = mass

↑
damping
constant

↑
spring
constant

⇒ all these questions are the
constant coeff case

determine - underdamped
- overdamped
- critically damped

write in form $A \sin(\omega t + \phi)$