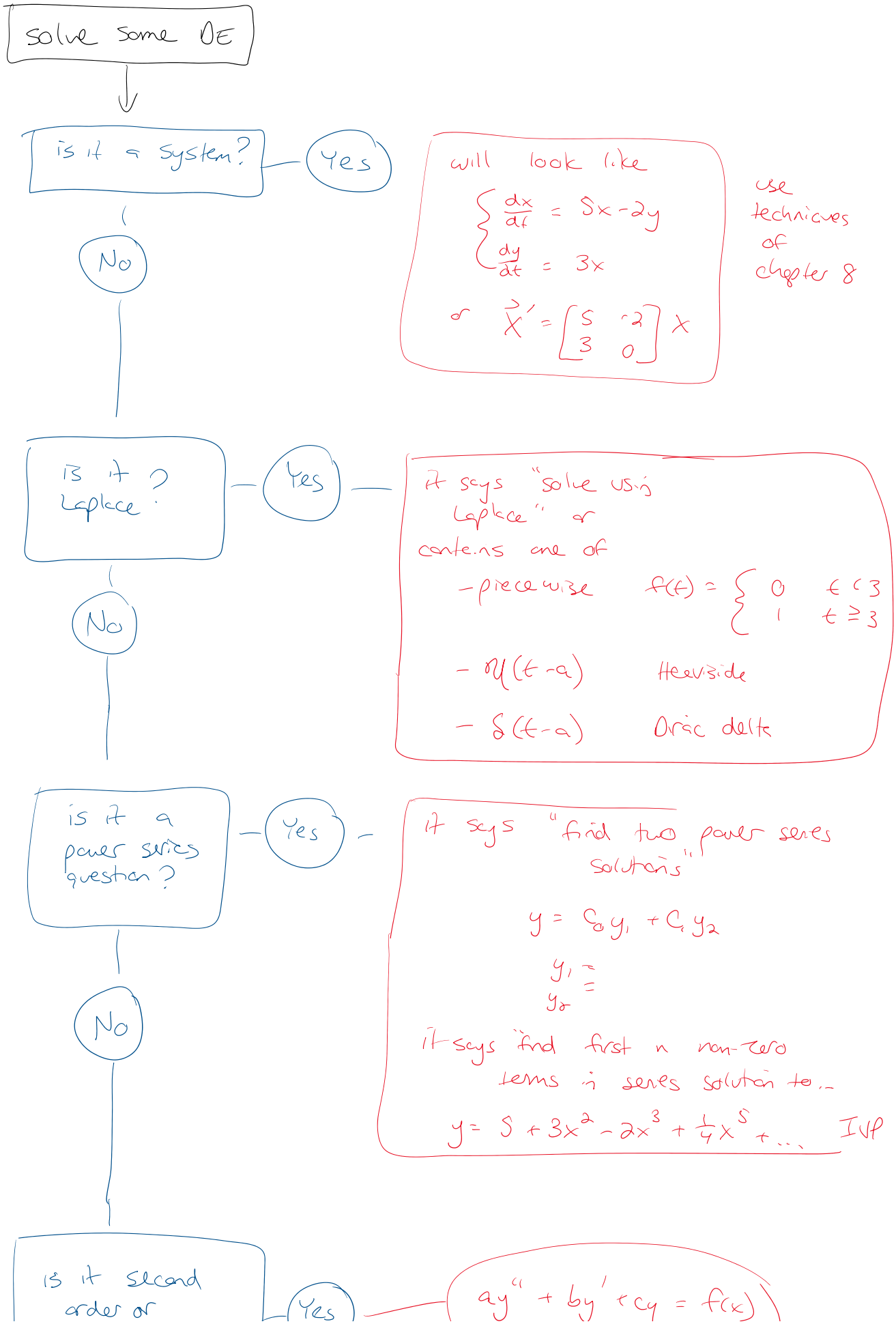
Decision Tree for solving Differential Equations

Tuesday, April 11, 2023 12:05 PM



is it second order or higher?

Yes

No

$ay'' + by' + cy = f(x)$
constant coeff

$ax^2y'' + bxy' + cy = f(x)$
Cauchy-Euler

are you given y_1? (first solution)
reduction of order

give y_c for homogeneous case

or y_c if non-homogeneous

but what to do about $f(x)$, the RHS, after you've found y_c?

if $ay'' + by' + cy = f(x)$ constant coeff and $f(x)$ contains only
- products of polynomials ($3x^2 + 2x$)
- and exponentials
- and sines/cosines

→ method of undetermined coeffs (watch for "bad case")

if const coeffs and $f(x)$ has
- $\ln x$
- $\tan x$, $\sec x$
- $\sqrt{x}$
- $1/x$

→ variation of parameters

if $ax^2y'' + bxy' + cy = f(x)$ Cauchy Euler

Is the DE first order?

yes

Separable

$f(x)\,dx = g(y)\,dy$

explicit solution – solve for dependent variable
if in x, y, then solve for y

implicit solution – don't bother to solve for y

linear 1st order

$\frac{dy}{dx} + P(x)\,y = f(x)$

integrating factor is

$IF = e^{\int P(x)\,dx}$

substitution →

Bernoulli

$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$

$\frac{dy}{dx} = f(Ax + By + C)$

$y' = \sin(3x+2y)$ $\quad \frac{dy}{dx} = \sqrt{5y-3x}$

homogeneous

homogeneous

$(x^3 + x^2 y)\,dy + (x^3 + xy^2 + y^3)\,dx = 0$

polynomial of degree 3

exact

$M(x,y)\,dx + N(x,y)\,dy = 0$

has $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

partial derivatives

- if not exact, can make it exact with IF

is it a word problem ?

1st order

proportionality

$$\frac{dy}{dt} = ky$$

examples: population growth

$$\frac{dP}{dt} = kP$$

k will be positive

exponential decay

$$\frac{dP}{dt} = kP$$

k will be negative

[or just do $\frac{dP}{dt} = -kP$ with k positive]

Newton's Law of Cooling

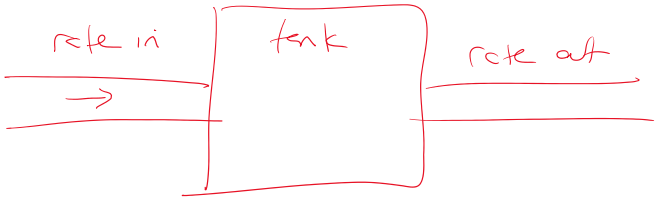
$$\frac{dT}{dt} = k(T - T_{room})$$

air resistence

$$\vec{F}_{drag} = k\vec{v}$$

↑ object's speed

mixture problems



$$\frac{dm}{dt} = \text{rate in} - \text{rate out}$$

↑ rate of change of mass of salt or other substance in tank

2nd order word problems

mass/spring system

external force ↓

$$\frac{md^2x}{dt^2} + \frac{b\,dx}{a} + kx = F_{ext}(t)$$

↑ m = mass

↑ damping constant

↑ spring constant

⇒ all these questions are the constant coeff case

determine
- underdamped
- overdamped
- critically damped

write in form $A\sin(\omega t + \phi)$