Review for Test 2:

Thursday, March 5, 2020 11:22 AM

Chapter 4: 2^{nd} order linear DEs (and higher order)

1. $y'' + P(x)\, y' + Q(x)\, y = 0$

when y_1 is known

find $$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

note: DE must be in std form with no coeffs on y'' term to get correct $P(x)$

2. $\underbrace{a y'' + b y' + c y}_{\text{solve homogeneous first}} = g(x)$ constant coeffs

$am^2 + bm + C = 0$

and write y_c

if $g(x) = 0$, $y = y_c$

if $g(x) =$ polynomial, exponential, sine/cosine, or product of these

MUC – guess y_p, then calculate coeffs

watch for "bad case"

otherwise, use variation of parameters

③ $\underbrace{ax^2 y'' + bxy' + cy}$ $= g(x)$ Cauchy-Euler

solve homogeneous first
$am(m-1) + bm + c = 0$
and write y_c

if $g(x) = 0$, $y = y_c$

if $g(x) \neq 0$, use var of parameters

2 distinct real, $y_c = c_1 x^{m_1} + c_2 x^{m_2}$
1 repeated real $y_c = c_1 x^m + c_2 x^m \ln x$
complex on formula sheet

GRAPHS

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variation of parameters:

$$y'' + P(x)y' + Q(x)y = f(x)$$

note: must be in std form to get $f(x)$ right

then write Ws, and integrate to get u_1 and u_2

$$y_p = u_1 y_1 + u_2 y_2$$

Chapter 5: Linear Models

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_{ext}$$

if no damping, $b = 0$

get $n = \pm i\sqrt{\frac{k}{m}} = \pm i\omega$

$$y_c = c_1 \cos \omega t + c_2 \sin \omega t$$

- could rewrite as $A \sin(\omega t + \phi)$

where $A = \sqrt{c_1^2 + c_2^2}$, $\tan \phi = \frac{c_1}{c_2}$

amplitude

ϕ = phase shift

period = $\frac{2\pi}{\omega}$

if damping, get

aux eqn: $mn^2 + bn + k = 0$

overdamped $\quad y_c = c_1 e^{n_1 t} + c_2 e^{n_2 t}$

critically damped $\quad y_c = (c_1 + c_2 t) e^{nt}$

underdamped $\quad y_c = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t)$

if external force, use MUC

method of undetermined coeffs