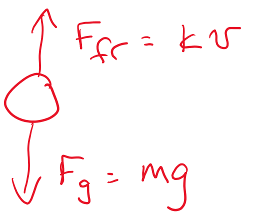
Section 1.1: Definitions

Tuesday, January 10, 2023 11:26 AM

example: Consider an object of mass m dropped from rest just above the earth's surface. The air resistence acting on the mass is proportional to the speed of the object through the air.



$$\sum \vec{F} = m\vec{a}$$

$$F_g - F_{fr} = ma$$

$$mg - kv = ma$$

$$mg - kv = m\frac{dv}{dt}$$

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see handout:

solution to DE examples:

c is a constant

(1) show that $y = c\ln x$ satisfies $y'\ln x - \frac{y}{x} = 0$

answer:

$$y = c\ln x$$
$$y' = \frac{c}{x}$$

sub back into DE:

$$y'\ln x - \frac{y}{x} = 0$$

$$\frac{c}{x}\ln x - \frac{c\ln x}{x} = 0$$

$$0 = 0 \quad \checkmark$$

② show that $y^3 - x^2 = 1$ is a solution to $\frac{dy}{dx} = \frac{2x}{3y^2}$

method #1: $y^3 = x^2 + 1$

$$y = \sqrt[3]{x^2+1} = (x^2+1)^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3}(x^2+1)^{-2/3} \cdot 2x$$

$$= \frac{2x}{3}(x^2+1)^{-2/3}$$

now sub back into DE:

$$\frac{dy}{dx} = \frac{2x}{3y^2}$$

$$\frac{2x}{3}(x^2+1)^{-2/3} = \frac{2x}{3(x^2+1)^{2/3}} \quad \checkmark$$

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method #2: implicit differentiation

$$y^3 - x^2 = 1$$

$$3y^2\frac{dy}{dx} - 2x = 0$$

now solve for $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{2x}{3y^2}$

now sub into DE: $\frac{dy}{dx} = \frac{2x}{3y^2}$

$$2x = 2x \quad \checkmark$$

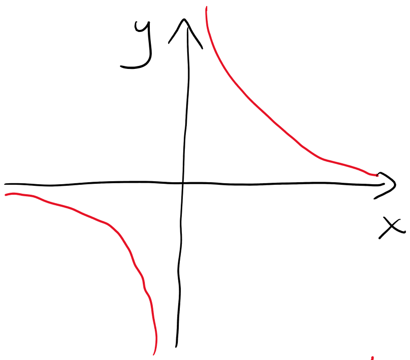
$3y^2 \quad 3y^2 \quad \checkmark$

Section 1.1 cont'd 2023/01/11

Interval of Solution :

- largest interval of x-values (indep variable) on which the solution is continuous

suppose a DE has solution $y = \frac{1}{x}$



continuous for $0 < x < \infty$

or

$-\infty < x < 0$

so the interval of solution is either

$-\infty < x < 0$

or

$0 < x < \infty$

(there may be several possible intervals)