Section 1.2: Initial-Value Problems (IVP)

Here's the big idea:

consider a DE like $\dfrac{dy}{dx} = f(x,y)$

$\uparrow$

slope

the solution is a family of curves in the xy plane

add in an initial condition, then you specify which of the curves it is

note: an nth order DE requires n initial conditions

example: consider the IVP $\begin{cases} y' + 2xy^2 = 0 \\ y(0) = -\dfrac{1}{4} \end{cases}$

a) This DE has solution $y = \dfrac{1}{x^2 + c}$.

Apply the initial condition to find the constant.

b) State the interval of solution

answer: a) at $x = 0$, $y = \dfrac{-1}{4} = \dfrac{1}{0^2 + c}$ so $c = -4$

then $y = \dfrac{1}{x^2 - 4}$

b) the function $y = \dfrac{1}{x^2 - 4}$ has denominator equal to

zero for $x = \pm 2$

possible intervals are

$x < -2$ or $\left(\!-2 < x < 2\!\right)$ or $x > 2$

but interval must contain initial condition $x = 0$

so interval is $\boxed{-2 < x < 2}$

initial conditions: we are given the value of
the dependent variables and/or
one or more of its derivatives
at a single value of the independent
variable

$\underset{dep}{y}$ vs $\underset{ind}{x}$

$\underset{dep}{x}$ vs $\underset{ind}{t}$