Section 2.4: Exact Equations

Friday, January 20, 2023 11:01 AM

background:

$$\text{if} \quad z = f(x,y)$$

$$\text{then} \quad dz = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy$$

partials dee partials dee

but how do you find a partial derivative? you pretend that the other variable is a constant and you differentiate

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now, if and only (iff)

$$f(x,y) = C \qquad \text{where } C \text{ is a constant}$$

then

$$\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy = 0$$

- what this means is if our DE

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

can be written in the form above, where

$$M = \frac{\partial f}{\partial x} \quad \text{and} \quad N = \frac{\partial f}{\partial y}$$

then we will just need to find f and set it to a constant

→ but how can we tell if we can write our DE in this form?

recall that for any continuous function

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

so if $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$ then ok ✓

and so $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then equation is exact

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example: state whether the following DEs are exact

(1) $(\sin y - y\sin x)\,dx + (\cos x + x\cos y - y)\,dy = 0$

$M(x,y) = \sin y - y\sin x$ | $N(x,y) = \cos x + x\cos y - y$

$\frac{\partial M}{\partial y} = \cos y - \sin x$ | $\frac{\partial N}{\partial x} = -\sin x + \cos y$

same! yes, exact