Section 4.1: Intro to Higher-order DEs

Tuesday, January 31, 2023 12:12 PM

examples of higher order linear DEs:

(1) $y'' - 9y = \underbrace{0}$ 2nd order linear homogeneous

RHS is zero ∴ homogeneous

(2) $x^2y'' - 5xy' + 4y = \underbrace{e^x}$ 2nd order linear non-homogeneous

(3) $y''' - y\sin x = \underbrace{\cos x}$ 3rd order linear non-homogeneous

note: $x^2dx + (3x^2 - y^2)dy = 0$

is homogeneous 1st order of degree 2

same word, different context

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general solution:

example: $y'' - 9y = 0$ has general solution

$$y = c_1e^{3x} + c_2e^{-3x}$$

where c_1 and c_2 are real constants

how can we tell that this is a solution? differentiate it twice and plus back into the DE

$$y = c_1e^{3x} + c_2e^{-3x}$$

$$y' = 3c_1e^{3x} - 3c_2e^{-3x}$$

$$y'' = 9c_1e^{3x} + 9c_2e^{-3x}$$

now sub back into DE:

$$y'' - 9y = 0$$

$$(9c_1 e^{3x} + 9c_2 e^{-3x}) - 9(c_1 e^{3x} + c_2 e^{-3x}) = 0 \quad \checkmark$$

this DE, $y'' - 9y = 0$ is 2nd order linear homogeneous

- has two arbitrary constants c_1, c_2
- 2 LI functions in solution, $y_1 = e^{3x}$, $y_2 = e^{-3x}$

example: (will not be tested)

consider the DE $y'' + y = 0$ with solutions $y_1 = \cos x$ and $y_2 = \sin x$

Are these solutions LI?

answer:

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x$$

$$= 1 \qquad \boxed{\neq 0}$$

$\boxed{\text{Yes}}$ because $W \neq 0$