Section 4.2: Reduction of Order

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standard form for a homogeneous 2^{nd} order linear DE:

$$y'' + P(x)\,y' + Q(x)\,y = 0$$

given one solution y_1,

you can find a second linearly independent solution:

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

then the general solution is:

$$y = c_1 y_1 + c_2 y_2$$

example: Consider the DE $y'' = 6y' - 9y$

with solution $y_1 = e^{3x}$.

a) show that y_1 is indeed a solution to the DE

b) find a second linearly independent solution

c) give the general solution

answer: a) $y_1 = e^{3x}$

$y_1' = 3e^{3x}$

$y_1'' = 9e^{3x}$

sub into DE: $y'' = 6y' - 9y$

$9e^{3x} = 6(3e^{3x}) - 9e^{3x}$

$9e^{3x} = 18e^{3x} - 9e^{3x}$ ✓

b) $y'' = 6y' - 9y$

rewrite in standard form:

$y'' - 6y' + 9y = 0$

std form: $y'' + P(x)\,y' + Q(x) = 0$

$P(x) = -6$

$\int P(x)\,dx = \int -6\,dx = -6x$ ← once again, don't bother with +C

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$= e^{3x} \int \frac{e^{6x}}{(e^{3x})^2} \, dx$$

$$= e^{3x} \int \frac{e^{6x}}{e^{6x}} \, dx$$

$$= e^{3x} \int dx$$

$$\boxed{y_2 = x e^{3x}}$$

c) $y = C_1 e^{3x} + C_2 x e^{3x}$

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example: find a second linearly independent solution to the following DE and then state the general solution. $y_1 = x^2$ is a sol'n.

$$y'' + \frac{2}{x} y' - \frac{6}{x^2} y = 0$$

answer: $P(x) = \frac{2}{x}$

$$\int P(x) \, dx = \int \frac{2}{x} \, dx = 2 \ln |x|$$

$$e^{-\int P(x)dx} = e^{-2\ln|x|}$$

$$= e^{\ln|x|^{-2}}$$

$$= \frac{1}{|x|^2}$$

$$= \frac{1}{x^2}$$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\, dx$$

$$= x^2 \int \frac{x^{-2}}{(x^2)^2}\, dx$$

$$= x^2 \int x^{-6}\, dx$$

$$= x^2 \left(\frac{-1}{5} x^{-5} \right)$$

$$= \frac{-1}{5} x^{-3} \quad \text{or just} \quad y_2 = x^{-3}$$

general solution: $y = C_1 y_1 + C_2 y_2$

$$y = C_1 x^2 + C_2 x^{-3}$$