

## Section 6.1: Review of Power Series

Wednesday, March 08, 2023 11:39 AM

see handout

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skill-builder: take two series written in sigma notation and rewrite them in the form of a single series

example.

$$\sum_{n=1}^{\infty} 5n C_n X^{n-1} + \sum_{n=0}^{\infty} 3 C_n X^{n+1}$$

answer: first, rewrite each series as a sum in  $X^k$

so let first series be in  $k = n-1$   
 $k+1 = n$   
and when  $n=1$ , then  $k=0$

so my first series becomes  $\sum_{k=0}^{\infty} 5(k+1) C_{k+1} X^k$

the second series is in  $k = n+1$   
 $k-1 = n$   
and when  $n=0$ ,  $k=1$

second series is now  $\sum_{k=1}^{\infty} 3 C_{k-1} X^k$

$$\begin{aligned} \text{recall, we want } & \sum_{n=1}^{\infty} 5n C_n X^{n-1} + \sum_{n=0}^{\infty} 3 C_n X^{n+1} \\ &= \sum_{k=0}^{\infty} 5(k+1) C_{k+1} X^k + \sum_{k=1}^{\infty} 3 C_{k-1} X^k \end{aligned}$$

but note that the two series don't have the same initial index ( $k=0$  for left-hand one,  $k=1$  for right)

so we need to make these match by rewriting the left series as

$$= \left[ 5 \cdot 1 \cdot c_1 \cdot x^0 + \sum_{k=1}^{\infty} 5(k+1)c_{k+1}x^k \right] + \sum_{k=1}^{\infty} 3c_{k-1}x^k$$

now combine to get

$$= 5c_1 + \sum_{k=1}^{\infty} \left[ 5(k+1)c_{k+1} + 3c_{k-1} \right] x^k$$

one other skill-builder:

let  $y = \sum_{n=0}^{\infty} c_n x^n$ , Find  $y'$  and  $y''$ .

answer:

$$y = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots + c_n x^n + \dots$$

$$y' = c_1 x^0 + 2c_2 x + 3c_3 x^2 + \dots + n c_n x^{n-1} + \dots$$

$$y'' = 2c_2 + 3 \cdot 2c_3 x + \dots + n(n-1)c_n x^{n-2} + \dots$$

$$\text{so } y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$