section 1.1 : Definitions

Monday, January 6, 2020 12:49 PM

see handout

Solution to a DE examples

c is constant

(1) show that $y = c\ln x$ satisfies $y' \ln x - \frac{y}{x} = 0$

answer: $y = c\ln x$

$y' = \frac{c}{x}$

sub back into DE:

$$y' \ln x - \frac{y}{x} = 0$$

$$\frac{c}{x} \ln x - \frac{c\ln x}{x} = 0$$

$$0 = 0 \quad \checkmark$$

(2) show that $y^3 - x^2 = 1$ is a soln to $\frac{dy}{dx} = \frac{2x}{3y^2}$

method #1:

solve for y: $y^3 = x^2 + 1$

$$y = \sqrt[3]{x^2+1} = (x^2+1)^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3}(x^2+1)^{-2/3} \cdot 2x$$

$$\frac{dx}{} = \frac{1}{3}(\quad) \quad = \frac{2x}{3}(x^2+1)^{-2/3}$$

sub back into DE: $\frac{dy}{dx} = \frac{2x}{3y^2}$

$$\frac{2x}{3}(x^2+1)^{-2/3} = \frac{2x}{3(x^2+1)^{2/3}}$$ ✓

---

method #2: $y^3 - x^2 = 1$

implicit differentiation: $3y^2\frac{dy}{dx} - 2x = 0$

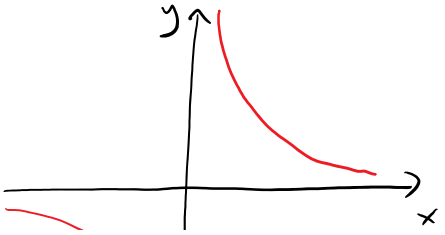
solve for $\frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{2x}{3y^2}$

sub into DE: $\frac{dy}{dx} = \frac{2x}{3y^2}$

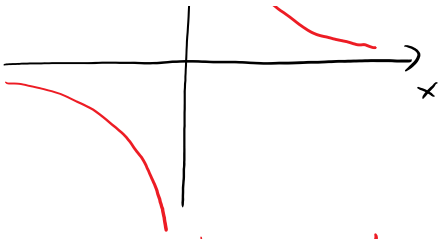
$\frac{2x}{3y^2} = \frac{2x}{3y^2}$ ✓

Interval of Solution:

- largest interval of x-values on which the solution is continuous



suppose a DE has solution $y = \frac{1}{x}$



continuous on $-\infty < x < 0$

continuous for $0 < x < \infty$

Interval : $-\infty < x < 0$ OR $0 < x < \infty$

(note: there may be several possible intervals)