Section 1.2: Initial-Value Problems (IVP)

Wednesday, January 8, 2020 12:25 PM

often we want to solve a DE subject to some constraints

example: at time $t=0$, the car is at rest at a certain location

- if we know the value of the dependent variable and/or one or more of its derivatives at a single value of the independent variable, these constraints are called

initial conditions

and the entire problem is called an

initial value problem (IVP)

note: if instead you know the dependent variable at two or more different values of the independent variable, problem is a

boundary-value problem (BVP)

example: beam with hinges at both ends

example: given that $y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-x}$ is a solution to the DE $2y'' + 3y' + y = 0$, calculate the values of the constants if

$$y(0) = 12 \quad \text{and} \quad y''(0) = 0$$

answer:

$$y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-x}$$
$$y' = -\tfrac{1}{2} c_1 e^{-\frac{1}{2}x} - c_2 e^{-x}$$
$$y'' = \tfrac{1}{4} c_1 e^{-\frac{1}{2}x} + c_2 e^{-x}$$

now sub in constraints:

at $x=0$, $y = 12 = c_1 + c_2$

$y'' = 0 = \frac{1}{4} c_1 + c_2$

this system has solution $c_1 = 16$ and $c_2 = -4$

so $\boxed{y = 16 e^{-\frac{1}{2}x} - 4 e^{-x}}$

here's the big idea:

consider a DE $\underbrace{\frac{dy}{dx}}_{\text{slope}} = f(x, y)$

the solution consists of a family of curves in the x-y plane

add an initial condition and you specify which of the curves it is

example: consider the IVP

$$\begin{cases} y' + 2xy^2 = 0 \\ y(0) = -1 \end{cases}$$

$\{ \; y(0) = -\frac{1}{4}$

a) Given that the DE has solution $y = \frac{1}{x^2+C}$,

solve this IVP.

b) State the interval of solution

a) sub in the initial condition:

at $x=0$, $y = -\frac{1}{4} = \frac{1}{0^2+C}$ so $C = -4$

so $\boxed{y = \frac{1}{x^2-4}}$

b) possible intervals are $x < -2$ or $-2 < x < 2$ or $x > 2$

but interval must contain the initial condition $x=0$

so interval is $\boxed{-2 < x < 2}$