Section 4.1: Intro to Higher Order Linear DEs

Thursday, January 30, 2020 11:30 AM

see handout

examples of higher order linear DEs:

① $y'' - 9y = 0$ — 2nd order linear, homogeneous

this is zero

② $x^2 y'' - 5xy' + 4y = e^x$ — 2nd order linear non-homogeneous

non-zero

③ $y''' - y\sin x = \cos x$ — 3rd order linear non homogeneous

note: $x^2\,dx + (3x^2 - y^2)\,dy = 0$

is homogeneous 1st order of degree 2

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general solution:

example: $y'' - 9y = 0$ has general solution

$$y = c_1 e^{3x} + c_2 e^{-3x}$$

where c_1 and c_2 are real constants

this DE, $y'' - 9y = 0$ is 2^{nd} order homogneous
2 arbitrary constants c_1, c_2
2 LI functions in solution,
$y_1 = e^{3x}$, $y_2 = e^{-3x}$

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example: consider the DE $y'' + y = 0$, with solutions $y_1 = \cos x$ and $y_2 = \sin x$

Are these solutions LI?

answer:

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x$$
$$= 1 \neq 0$$

$\boxed{\text{Yes}}$ because $W \neq 0$

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one last thing:

notation : differential operator D

$$\frac{d}{dx} = D \quad \text{so that} \quad \frac{dy}{dx} = Dy$$

$$\frac{d^2y}{dx^2} = D^2y$$

and $y'' - 2y' - 8y = 0$

becomes

$$D^2y - 2Dy - 8y = 0$$

$$(D^2 - 2D - 8)\,y = 0$$

$$(D-4)(D+2)\,y = 0$$