Section 6.1: Review of Power series

Tuesday, March 3, 2020 10:26 AM

see handout

skill-builder: how to rewrite the sum of two series in sigma notation into the form of a single series

example: $\sum_{n=1}^{\infty} 5n\, c_n\, x^{n-1} + \sum_{n=0}^{\infty} 3\, c_n\, x^{n+1}$

answer: first, rewrite each series as a sum in x^k

$$\sum_{n=1}^{\infty} 5n\, c_n\, x^{n-1}$$

let $k = n-1$

$k+1 = n$

when $n=1$, $k=0$

$$= \sum_{k=0}^{\infty} 5(k+1)\, c_{k+1}\, x^k$$

similarly, $\sum_{n=0}^{\infty} 3\, c_n\, x^{n+1}$

let $k = n+1$

$k-1 = n$

when $n=0$, $k=1$

$$= \sum_{k=1}^{\infty} 3\, c_{k-1}\, x^k$$

then we say

$$\sum_{k=0}^{\infty} 5(k+1)\, c_{k+1}\, x^k + \sum_{k=1}^{\infty} 3\, c_{k-1}\, x^k$$

$$\sum_{k=0} 5(k+1)c_{k+1}\, x \quad + \quad \sum_{k=1} 3\, c_{k-1}\, x$$

$$\left(5\cdot 1\cdot c_1 x^0 + \sum_{k=1}^{\infty} 5(k+1)c_{k+1} x^k\right) + \sum_{k=1}^{\infty} 3\, c_{k-1}\, x^k$$

now rewrite:

$$5c_1 + \sum_{k=1}^{\infty}\left[5(k+1)c_{k+1} + 3c_{k-1}\right]x^k$$

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one other skill-builder:

let $y = \sum_{n=0}^{\infty} c_n x^n$. find y' and y''.

$$y = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1} = c_1 + 2c_2 x + 3c_3 x^2 + \dots$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} = 2c_2 + 3\cdot 2 c_3 x + \dots$$