Math 252 – Test 2: Version A

March 6, 2020 **Name:** _____

Instructor: Patricia Wrean

Total: 25 points

1. (4 points) Find a second solution to the following DE, given that $y_1 = \cos(x^2)$ is a solution. You may assume that $x > 0$.

$$xy'' - y' + 4x^3y = 0$$

2. (4 points) Consider the following DE.

$$x^2y'' + 5x\,y' + cy = 0$$

Solve this DE for the following values of c.

(a) $c = 4$

(b) $c = 5$

3. (6 points) Use the method of variation of parameters to solve the following DE.

$$y'' + y = \sec x$$

4. (6 points) Consider the following differential equations and graphs.

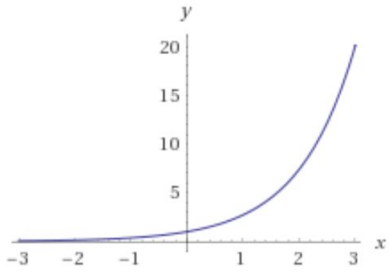
   (a) State the form of the particular solution y_p for the following. Leave your answer with undetermined coefficients. Please note that the complementary solution for the homogeneous equation is $y_c = C_1 e^{-2x} + C_2 e^{-4x}$.

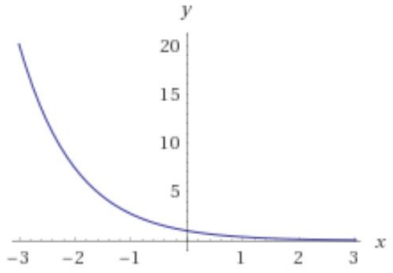
   (i) $y'' + 6y' + 8y = 5e^{-2x}$ _____

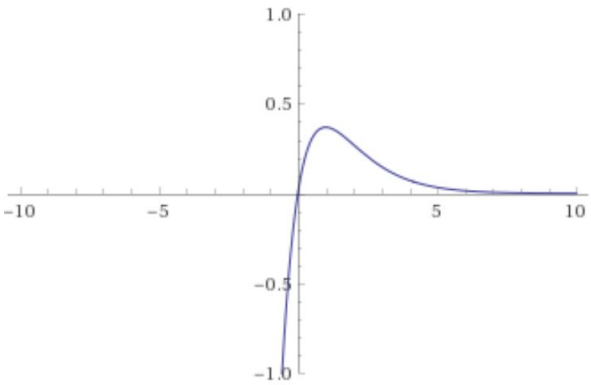
   (ii) $y'' + 6y' + 8y = e^x + \cos(2x)$ _____

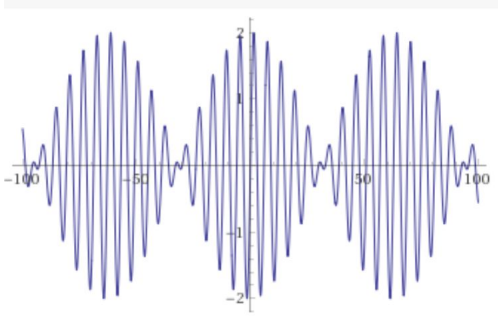
   (iii) $y'' + 6y' + 8y = e^x \cos(2x)$ _____

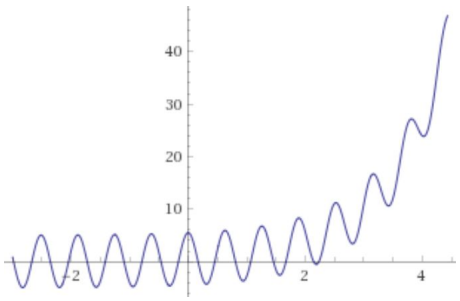
   (b) For each of the solutions y_p above, indicate which plot or plots below are possible graphs of that y_p. No explanation is required. You may pick more than one.

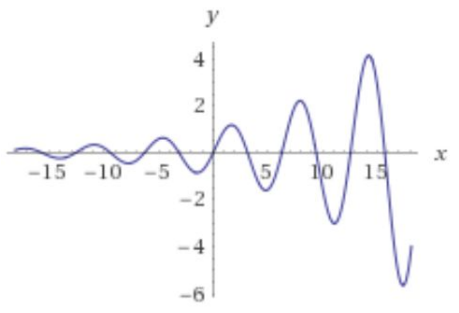












5. (5 points) A mass of 0.5 kg is attached to a spring with constant 12.5 N/m. There is a damping force such that the damping constant is numerically equal to 5. The mass is released from 0.3 m above equilibrium with a downward velocity of 0.6 m/s.

   (a) Find the position $y(t)$ of the mass as a function of time.

   (b) This motion is: (choose one)

   (i) overdamped

   (ii) critically damped

   (iii) underdamped