Term: Winter, 2021 Name: Solution Set

Instructor: Patricia Wrean

Math 252-DX02
Test 1

Total = $\frac{\quad}{20}$

- **Show your work. All of the work on this test must be your own.** While writing this test, you may not consult any other person, website, or other resource not listed below. If you have a question during the test, you may email me.
- Here is a list of the resources that you are allowed to use during this test:
  - your own notes
  - lecture notes, videos, handouts, practice questions, and practice tests from either my website at http://wrean.ca/math252 or the Math 252 websites of any of the other instructors linked on the landing page of my site
  - your textbook (Zill), or any of the texts listed on the Textbook page at http://wrean.ca/math252/math252_textbook.htm
  - the Math 252 D2L website
  - the Math 252 WeBWorK online homework site
  - a scientific calculator. You may not use a calculator with graphing capability. If you like, you may use a scientific calculator app like the one at Desmos: https://www.desmos.com/scientific
  - a handy reference is the Math 252 Formula Sheet at http://wrean.ca/math252/tests/math252_formula.pdf
  - if you have questions during the test, you may email me
- To submit this test, please use the Dropbox feature in the Assignments tab of D2L. Please assemble your answers into a single PDF or Word document, unless you've made other arrangements with me beforehand. Helpful software:
  - Genius Scan app at https://www.thegrizzlylabs.com/genius-scan/
  - CombinePDF at https://combinepdf.com/

GOOD LUCK!

1. (5 points) Consider the following DE.

$$y' + 4xe^y = e^y \cos x$$

(a) Solve this DE, giving an explicit solution.

(b) Is your answer to part (a) a particular solution? Circle one: Yes / No (1)

because there is an arbitrary constant C_1

$y' = e^y \cos x - 4xe^y$

$\frac{dy}{dx} = e^y(\cos x - 4x)$ (1) separable (1)

$\int e^{-y}\,dy = \int (\cos x - 4x)\,dx$

$-e^{-y} = \sin x - 2x^2 + C$ (1)

solve explicitly.

$e^{-y} = -\sin x + 2x^2 - C$

$y = -\ln(-\sin x + 2x^2 - C)$

$y = -\ln(2x^2 - \sin x + C_1)$ (1)

2. (5 points) Consider the following DE. You may assume that $x > 0$.

$$\left(\frac{2y^2}{x} + 6x\right) dx = k(1 - y \ln x) dy$$

(a) Find the value of k for which this DE is exact.

$\underbrace{\left(\frac{2y^2}{x} + 6x\right) dx}_{M} \quad \underbrace{- \; k(1 - y \ln x) dy}_{N} = 0$

$\frac{\partial M}{\partial y} = \frac{4y}{x}$ $\qquad$ $\frac{\partial N}{\partial x} = \frac{ky}{x}$

so $\boxed{k = 4}$ (1)

(1)

(b) Solve this DE using the value of k you found in part (a).

$\left(\frac{2y^2}{x} + 6x\right) dx - 4(1 - y \ln x) dy = 0$

$f = \int \left(\frac{2y^2}{x} + 6x\right) dx$ and $f = \int -4(1 - y \ln x) dy$

$= 2y^2 \ln x + 3x^2 + g(y)$ $\qquad$ $f = -4y + 2y^2 \ln x + h(x)$

(1) (1)

$f = 2y^2 \ln x + 3x^2 - 4y = C$

$\boxed{2y^2 \ln x + 3x^2 - 4y = C}$ (1)

3. (5 points) Solve the following DE.

$$y' = y + \frac{6e^{-x}}{y}, \qquad y(0) = 2$$

$y' - y = 6e^{-x} y^{-1}$ (1) Bernoulli with $n = -1$

so let $v = y^{1-n} = y^2$, $\frac{dv}{dx} = 2y \frac{dy}{dx}$ (1)

↑ mult both sides by this

$2yy' - 2y^2 = 12e^{-x}$

$\frac{dv}{dx} - 2v = 12e^{-x}$ (1)

linear 1st order

$IF = e^{\int P(x)\,dx} = e^{\int -2\,dx} = e^{-2x}$

$\frac{dv}{dx} e^{-2x} - 2ve^{-2x} = 12e^{-3x}$

$\frac{d}{dx}(ve^{-2x}) = 12e^{-3x}$

$\int \frac{d}{dx}(ve^{-2x})\,dx = \int 12e^{-3x}\,dx$

$ve^{-2x} = -4e^{-3x} + C$

$y^2 e^{-2x} = -4e^{-3x} + C$ (1)

but at $x = 0$, $y = 2$

$4e^0 = -4e^0 + C$ so $C = 8$

$$y^2 e^{-2x} = -4e^{-3x} + 8$$ (1)

4. (5 points) After washing your clothes, you are hanging them up to dry. You think that the damp clothes lose their water at a rate proportional to the amount of water still present in the clothes. If after 45 minutes, the clothes have 65% of their original water content left, after how long will half of the water have evaporated?

   Start by setting up and solving the associated DE. Show your work.

let W = amount of water present in clothes

$\frac{dW}{dt} = kW$ separable (1)

$\int \frac{dW}{W} = \int k\,dt$

$\ln|W| = kt + C$ but W is never negative so can drop $|\ |$

$\ln W = kt + C$

$W = e^{kt+C} = e^{kt}e^{C}$

$\boxed{W = C_1 e^{kt}}$ (1)

if you like, you can say that at $t = 0$, $W = W_0$

then $\boxed{W = W_0 e^{kt}}$

after 45 minutes, $W = 0.65\,W_0$

$0.65\,W_0 = W_0 e^{k\cdot 45}$

$0.65 = e^{45k}$

$\ln 0.65 = 45k$

$k = \frac{1}{45}\ln(0.65)$ (1)

need to find t when $w = \frac{1}{2} w_0$:

$$\frac{1}{2} w_0 = w_0 e^{kt} \qquad (1)$$

$$\frac{1}{2} = e^{kt}$$

$$\ln\left(\frac{1}{2}\right) = kt$$

$$t = \frac{1}{k} \ln\left(\frac{1}{2}\right)$$

$$= 45 \, \frac{\ln\left(\frac{1}{2}\right)}{\ln(0.65)}$$

$$\approx 72.4068$$

$t \approx 72$ minutes (1)