Section 4.1: Intro to Probability

Tuesday, November 5, 2019 10:46 AM

in statistics, probabilitity is used as a tool to evaluate the reliability of conclusions about a population based on a sample

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experiment - process by which an observation (measurement) is obtained

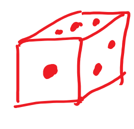
simple event - the outcome observed on a single repetition of an experiment

compound event - a collection of simple events (sometimes just called an event)

examples: ① flipping a coin ← experiment

simple events: heads, tails

② rolling a 6-sided die



↑ die is singular
dice is plural

simple events: 1, 2, 3, 4, 5, 6

compound event of rolling an even number:

$\{2, 4, 6\}$

mutually exclusive: two events are mutually exclusive if, when one event occurs, the other event cannot occur

example: rolling a 6-sided die

are the following pairs of events mutually exclusive?

a) $\begin{cases} \text{rolling an odd number} \\ \text{rolling a 2} \end{cases}$ ✓

note: mutually exclusive events don't have to "span the sample space" – there can be other events left over

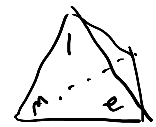
what is the sample space? it is a list of all simple events (all possible outcomes)

b) $\begin{cases} \text{rolling a 1 or a 2} \\ \text{rolling } \geq 2 \end{cases}$ ✗

note: simple events are always mutually exclusive!

if you've rolled a 2, you haven't rolled any other value example: rolling a pair of 4-sided dice

what is the sample space?



Sample space:

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

list of all possible outcomes

a total of 16 simple events

note: if the two dice are fair, then there is an equal chance of landing on any of the sides. In this case, the probability of any simple event is $\frac{1}{16}$

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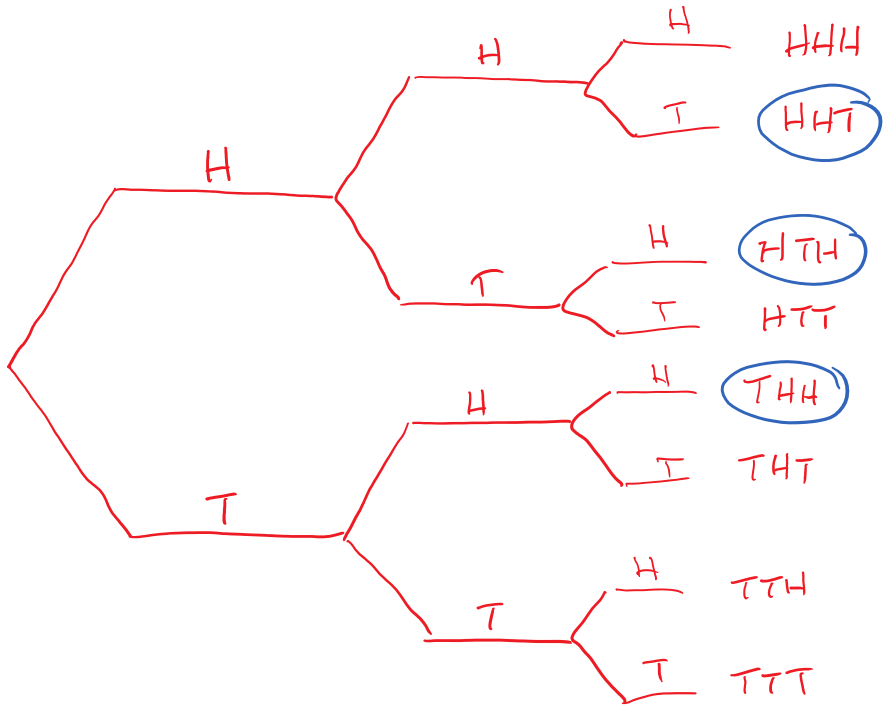
if you are having trouble generating the sample space, another approach is a tree diagram:

example: write out the sample space for flipping a coin 3 times and recording the result



note: how many ways can you get only one tail?

if the coin is fair, then the probability of getting exactly one tail is 3/8

2019/11/06

up until now, we have figured out the size of the sample space by writing out the full list of outcomes

- which doesn't work as well when the sample space is <u>large</u>

<u>counting techniques</u>

example: How many 4-digit positive integers are evenly divisible by 5?

answer: 1000, 1005, 1010, ... 9995

note: this is an arithmetic sequence with $d=5$ from Math 155

new method:

number of choices for each digit: $\underset{\text{choose from } 1-9}{\underline{9}}$ $\underbrace{\underline{10} \quad \underline{10}}_{\text{choose from } 0-9}$ $\underset{\text{choose from } 0 \text{ or } 5}{\underline{2}}$

now multiply together to get

$$\# \text{ choices} = 9 \cdot 10 \cdot 10 \cdot 2 = 1800$$

note: this method only works when you can rule out possibilities in one or more slots

- so you can use it for "divisible by 5 or 2 or 10"

but you cannot use it for "divisible by 3 or 7", for example

multiplication rule:

suppose we have an event which is made up of n different independent steps

then times

then

times

total number of ways the event can happen = $\underline{\quad\quad} \times \underline{\quad\quad} \times \underline{\quad\quad} \times \ldots \times \underline{\quad\quad}$

number of ways the first step can happen

and you multiply these numbers together

note: what does "independent" mean?

the result of any choice does not change the number of choices for later steps

example: How many different BC licence plates for cars are there?
(assume all letters and numbers are used and ignore reserved words and personalized plates)

patterns:

LLL NNN
NNN LLL
LLN NNL

N = number
L = letter

answer: top pattern

$\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\quad \underset{N}{\underline{10}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}}$

number plates with this pattern $= 26^3 \cdot 10^3$

$= 17\ 576\ 000$

total number for all patterns: $3(17\ 576\ 000)$
$= 52\ 728\ 000$

example: In the mythical Canadian province of Gondor, licence plates follow the pattern letter - letter - letter - number - number. Due to recent political events, the letter combination EYE is no longer allowed. How many legal licence plates are there in Gondor?

answer:

number of legal plates = total number − number of illegal plates

total number: $\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}} = 26^3 \cdot 10^2$

illegal plates: $\underset{E}{\underline{1}}\ \underset{Y}{\underline{1}}\ \underset{E}{\underline{1}}\ \underset{N}{\underline{10}}\ \underset{N}{\underline{10}} = 10^2$

number of legal plates $= 26^3 \cdot 10^2 - 1^3 \cdot 10^2$
$= 1\ 757\ 500$

note: the reason you can't just say

25 25 25 10 10

is that ECQ 58 is still okay - can still start with an E in some cases

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tip: when finding the number of allowable outcomes sometimes it's easier to calculate the total number of outcomes and subtract the number of outcomes that are not allowed.

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the addition rule:

example: how many positive integers from 1 to 20 inclusive are

a) evenly divisible by 2?
b) 3?
c) 2 or 3?

answer: brute force method:

divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 — 10

divisible by 3: 3, 6, 9, 12, 15, 18 — 6

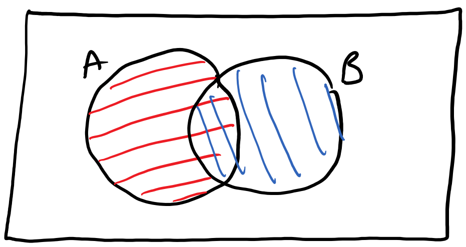
divisible by 2 or 3: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 — 13 note $13 \neq \underbrace{10 + 6}$

contains duplicates
6, 12, 18 appear in both lists

so $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

number of events containing A or B

can write as $n(AB)$



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example: How many 4-digit PINs

a) start with a 9?

b) end in a 4?

c) start with 9 or end in 4?

d) start with 9 or 4?

a) start with 9: $\underline{1}\ \underline{10}\ \underline{10}\ \underline{10} = 1000$

b) end with 4: same

c) start with 9 and 4: $\underline{1}\ \underline{10}\ \underline{10}\ \underline{1} = 100$

$$\begin{aligned} n(\text{start 9 or end 4}) &= n(\text{start 9}) + n(\text{end 4}) - n(\text{both}) \\ &= 1000 + 1000 - 100 \\ &= 1900 \end{aligned}$$

d) $$\begin{aligned} n(\text{start 9 or 4}) &= n(\text{start 9}) + n(\text{start 4}) - n(\text{both}) \\ &= 1000 + 1000 - 0 \\ &= 2000 \end{aligned}$$

or $\underline{2}\ \underline{10}\ \underline{10}\ \underline{10} = 2000$

2019/11/07

example: How many 4-digit PINs are there if repetition of digits is not allowed?

answer: $\underline{10}\ \underline{9}\ \underline{8}\ \underline{7} = 5040$

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digression: will not be tested

there's! another! way! to! calculate! this!

$$10 \cdot 9 \cdot 8 \cdot 7 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{6!}$$

where number of choices = 10
number of slots = 4

$(10-4)!$
is the denominator

this is called a permutation and on your calculator, it's the nPr button, so enter $_{10}P_4$ to get 5040

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example: How many 5-digit case-sensitive alphanumeric psswords are there

a) in total
b) that contain at least one letter and at least one number?

a) alphanumeric - characters can be either letters or numbers

case-sensitive - uppercase and lower case letters are considered to be different (case matters) so "A" is different than "a"

$$\underline{62}\ \underline{62}\ \underline{62}\ \underline{62}\ \underline{62} = 62^5 = 916\ 132\ 832$$

how many choices? $62 = 26 + 26 + 10$

uppercase letter, lower case, digits

b) total allowed = total possible - total not allowed all letters: $\underline{52}\ \underline{52}\ \underline{52}\ \underline{52}\ \underline{52} = 52^5$

all numbers: 10^5

total allowed: $62^5 - 52^5 - 10^5$

$= 535\ 828\ 800$