Section 4.2: Classical Probability

Thursday, November 7, 2019 9:49 AM

classical probability: if all outcomes are equally likely, then the probability of event E happening is

$$\underbrace{P(E)}_{\text{probability of } E \text{ happening}} = \frac{n(E)}{n_{tot}}$$

where $n(E)$ = number of ways E can happen

n_{tot} = total number of outcomes

example: If you roll two fair 4-sided dice, what's the probability that the sum of the rolls is 3 or less?

fair = all rolls are equally likely (loaded = not fair)

sample space:

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

16 outcomes in total

$$P(\text{sum} \le 3) = \frac{n(\text{sum} \le 3)}{n_{tot}}$$

$$= \frac{3}{16}$$

What, then, is the probability that the sum will be greater than 3?

$$P(\text{sum} > 3) = 1 - P(\text{sum} \le 3)$$

$$= 1 - \frac{3}{16}$$

$$= \frac{13}{16}$$

Why? $P(A) = 1 - P(\bar{A})$

What is the probability of rolling a sum of 5?

$$P(\text{sum} = 5) = \frac{n(\text{sum} = 5)}{n_{tot}} = \frac{4}{16} = \frac{1}{4}$$

What is the probability that at least one die shows the number 3?

$$P(\text{at least one } 3) = \frac{7}{16}$$

for event A, the complement can be written as

$$\bar{A}$$

the complement of A is the set of simple events in which A does not occur

$$P(A) + P(\bar{A}) = 1 \text{ or } 100\%$$

other notations: A^c

A'

$\sim A$

$\neg A$

two properties of probability:

(1) $0 \leq P(A) \leq 1$

(2) the sum of probabilities for all possible outcomes is 1

$$\sum_i P(A_i) = 1$$

example: At the Red Barn Market, you can get an ice-cream cone with two scoops of ice cream. Let's assume that you have to choose two different flavours for your scoops and that which flavour is on top doesn't matter. Let's further assume that when averaged over all customers, each flavour is equally likely

flavours available are: chocolate
vanilla
strawberry
bubble gum

a) How many different ice cream cones are possible?

brute force: CV VS SB
CS VB
CB

6 different cones

b) What's the probability that a random customer will order chocolate as one of the two scoops?

$$P(C) = \frac{n(C)}{n_{tot}} = \frac{3}{6} = \frac{1}{2} \text{ or } 50\%$$

c) what's the probability that a random customer will order chocolate and vanilla?

$$P(CV) = \frac{n(CV)}{n_{TOT}} = \frac{1}{6}$$

d) what's the probability that a random customer will order chocolate or vanilla?

$$P(C\text{ or }V) = \frac{n(C\text{ or }V)}{n_{TOT}} = \frac{5}{6}$$

e) Calculate d) again using a different method

$$P(C\text{ or }V) = 1 - P(\overline{C\text{ or }V})$$

$$= 1 - P(SB)$$

$$= 1 - 1/6 = 5/6$$

f) calculate d) again using <u>yet</u> <u>another</u> <u>method</u>!

$$P(C\text{ or }V) = P(C) + P(V) - P(CV)$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{6}$$

$$= \frac{5}{6}$$

2019/11/13

summary up until now:

$$P(\text{event}) = \frac{n(\text{event})}{n_{tot}}$$

$$P(\text{event}) = 1 - P(\overline{\text{event}})$$

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

this term can be zero
– then the events are said
to be "mutually exclusive"

note: for questions in which the sample space is small, the brute force method of writing out the complete sample space is a perfectly acceptable method

---

contingency table:

for classical probability, it is assumed that all outcomes are equally likely

in reality, this is frequently not true

one way of summarizing this kind of situation is called a contingency table see handout

conditional probability: it is the probability of event B happening if event A has happened or is happening

note: there does not have to be a causal link: A does not necessarily cause B to happen or vice versa

for example: you might be more likely to carry an umbrella if it is raining

$$P(\text{umbrella} \mid \text{raining}) = 75\%$$

if

"prob of umbrella if raining"

$$P(\text{umbrella} \mid \overline{\text{raining}}) = 0\%$$

note: if you do not own an umbrella, you might have

$$P(U \mid R) = 0\%$$
$$P(U \mid \bar{R}) = 0\%$$

and we say (more on this later) that these two events in this circumstance are independent – the probability of your carrying an umbrella does not depend on whether it is raining

---

so, how do you calculate conditional probabilities?

$$P(B \mid A) = \frac{P(AB)}{P(A)} = \frac{n(AB)}{n(A)}$$

---

see handout again

---

independent vs. dependent variables:

consider two events A and B:

- if B is just as likely to occur when you look at the entire population as when you look at subpopulation A, then these events are said to be independent

examples: you are rolling a fair six-sided die

P(rolling a 2) = $\frac{1}{6}$

P(rolling a 2 | rolled an even number) = $\frac{1}{3}$

so the probability of rolling a 2 depends on whether the roll was even

now let's go the other way:

P(rolling an even number) = $\frac{1}{2}$

P(rolling an even number | rolled a 2) = 100% or 1

what's important here is that these two numbers are not the same

so how do you determine whether two events are independent? can do any of:

1. calculate P(B) and P(B|A)

   if P(B) = P(B|A), then independent

2. calculate P(A) and P(A|B)

   if P(A) = P(A|B), then independent

or, if you insist,

(3) calculate $P(A \mid B)$ and $P(A \mid \bar{B})$

if equal, independent

(4) calculate $P(AB)$, $P(A)$, and $P(B)$

if $P(AB) = P(A) \cdot P(B)$, then independent