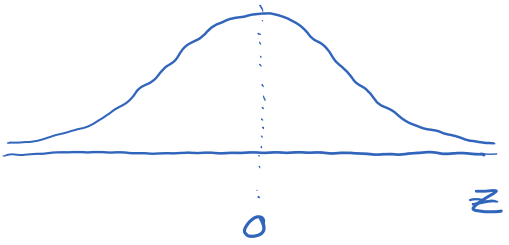
Section 5.3: The Standard Normal Distribution: Computing probabilities from z-scores

Monday, November 18, 2019 11:05 AM

the standard normal distribution:



where $z = \frac{x - \mu}{\sigma}$ the number of standard deviations away from the mean the point x is

this graph is perfectly symmetrical and the area under the curve is exactly equal to one

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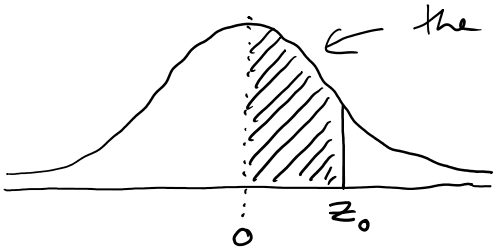
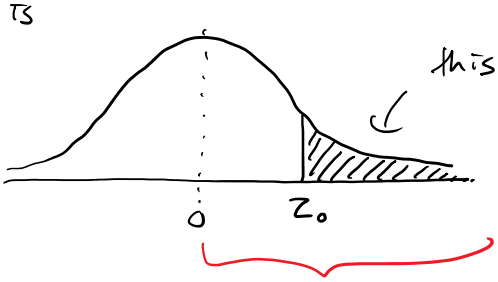
so if

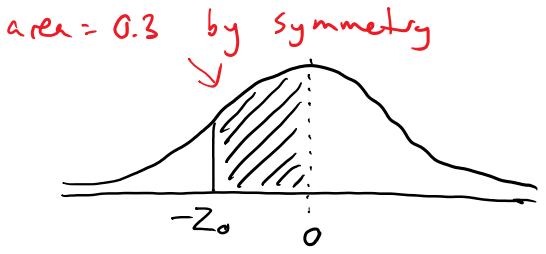


what, then is

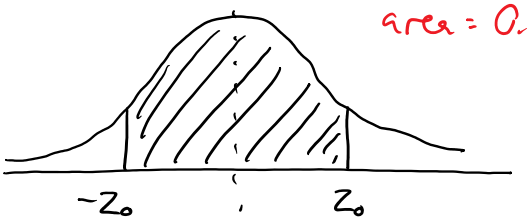


this entire area = 0.5

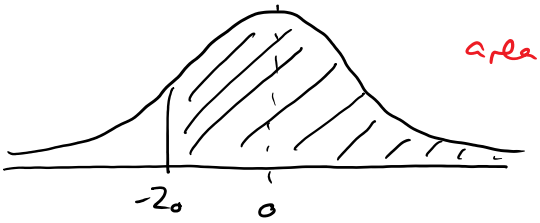
and



and



and

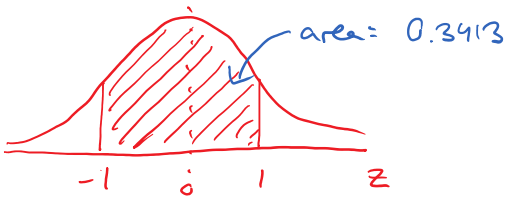


2019/11/19

What is the probability that a normally distributed data point is within

a) one standard deviation of the mean?
b) two " " " " " ?

answer: a)



$P(-1 < z < 1) = 2(0.3413)$

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$= 0.6826$ or **68.26%**

b) $P(-2 < z < 2) = 2(0.4772) = 0.9544$

or **95.44%**

Oh, look! It's the Empirical Rule!

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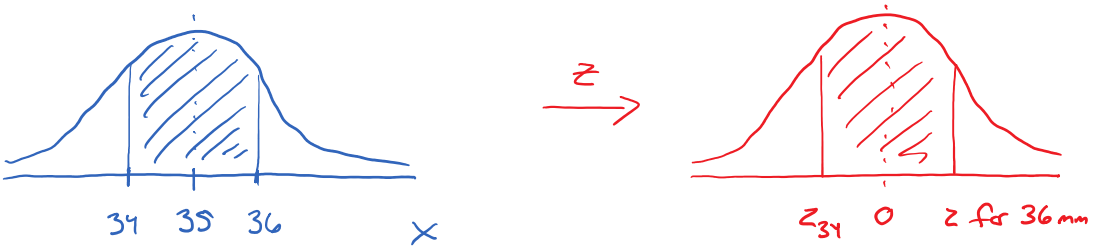
example: A botanist is studying the growth of a certain type of tomato plant. She finds that under certain growing conditions that the diameter is normally distributed with an average of 35 mm and a standard deviation of 3 mm.

$\mu = 35$ mm $\quad \sigma = 3$ mm

a) What is the probability that a particular tomato plant has a diameter between 34 and 36 mm?



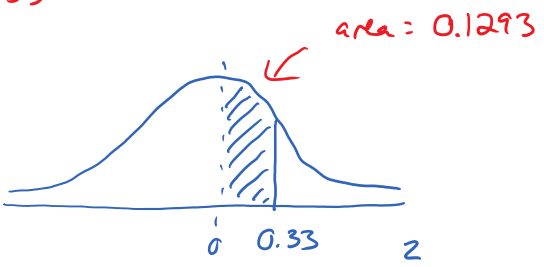
for 36 mm: $z_{high} = \frac{x - \mu}{\sigma} = \frac{36 - 35}{3}$

$= \frac{1}{3}$ or $0.\overline{33} \approx 0.33$

(on the table, the best we can do

(3 two decimal places)

for 34 mm, $z_{low} \approx -0.33$



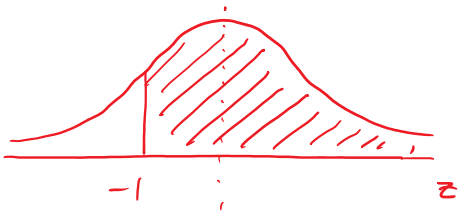
$$P = 2(0.1293)$$
$$= 0.2586$$

$$= \boxed{26\%}$$

note: the original problem did not have a lot of precision (did not have many decimal places), so it's appropriate to round a bit here

b) What's the probability that a random tomato plant has a diameter greater than 32 mm?

$$z = \frac{x - \mu}{\sigma} = \frac{32 - 35}{3} = -1$$



area = 0.3413 | 0.5 by symmetry

$$P = 0.3413 + 0.5$$
$$= 0.8413$$

$$= \boxed{84\%}$$